High-field noise in degenerate and mesoscopic systems

F. Green[†]
GaAs IC Prototyping Facility Program, CSIRO Telecommunications and Industrial Physics, PO Box 76, Epping NSW 1710, Australia

M. P. Das[‡]
Department of Theoretical Physics, Research School of Physical Sciences and Engineering, The Australian National University, Canberra ACT 0200, Australia

We analyse high-field current fluctuations in metallic systems by direct mapping of the Fermi-liquid correlations to the semiclassical nonequilibrium state. We give three applications. First, for bulk conductors, we show that there is a unique nonequilibrium analogue to the fluctuation-dissipation theorem for thermal noise. With it, we calculate suppression of the excess hot-electron term by Pauli exclusion. Second, in the degenerate mesoscopic regime, we argue that shot noise and thermal noise are incommensurate. They cannot be connected by a smooth, universal interpolation formula. This follows from their contrasting responses to Coulomb screening. We propose an experiment to test this mismatch. Third, we carry out an exact model calculation of high-field shot noise in narrow mesoscopic wires. We show that a distinctive mode of suppression arises from the structure of the Boltzmann equation in two and three dimensions. In one dimension such a mode does not exist.

I. INTRODUCTION

High-field noise in degenerate conductors still lacks a systematic theoretical description, despite its importance for microelectronics.[1] Here we advance a practicable theory of nonequilibrium fluctuations in metallic systems, accounting for the dominant Fermi-liquid behaviour of the electrons.[2] There is no electronic property of a metal near equilibrium that is not governed by Pauli exclusion, from the microscopic level to the bulk.[3] In this paper we analyse how degeneracy determines nonequilibrium current noise over a wide range of length scales, even at high fields.

Technological developments of late have led to a variety of delicate measurements of transport and noise in many different mesostructures.[4–10] Alongside the experiments there has been much theoretical activity.[11–17] A particular topic, still being elaborated, is the behaviour of current fluctuations in mesoscopic conductors. These are typically shorter than the inelastic mean free path but longer than that for elastic processes. They are in the regime of diffusive transport, where two diverse understandings predominate. One technique is the quantum-transmission (Landauer) method[18] as applied to fluctuations by Lesovik,[12] Beenakker and Büttiker,[13,14] Martin and Landauer,[15] and many others.[11] The second approach is the semiclassical-transport (Boltzmann) method[19] associated with Nagaev[16] and de Jong and Beenakker.[11,17] Both view a mesoscopic wire as an assembly of individual elastic scatterers in a bath of free carriers. The formalisms, however, are quite dissimilar.[20]

In the Landauer model, multiple scattering preserves the coherence of single-particle propagation; the only way in which current fluctuations can lose correlation strength is by interplay of the transmission amplitudes and nonlocal Pauli blocking between state occupancies in the source and drain leads.[21] In the Boltzmann model, incoherence enters from the start through the Stosszahlansatz. To study the fluctuations, the stochastic collision term is supplemented with a set of Langevin flux sources whose phenomenology is that of classical shot noise;[19] their self-correlations reflect the sporadic timing of elementary encounters between discrete wave packets and scatterers. The induced fluctuations lose correlation strength when diffusive elastic scattering is locally modified by Pauli exclusion.[11,22,23]

Regardless of their differences, both methods can explain observations such as the threefold suppression of shot noise when elastic scattering prevails.[6] Because the phase-coherent model is fully quantum mechanical, its fluctuation structure is natural, not imposed, and its predictions enjoy a definitive status. On the other hand, semiclassical phenomenology is the more natural tool when inelastic (hence irreducibly phase-breaking) collisions are important, as in high-field transport.

Our paper investigates fluctuations beyond the elastic weak-field limit. A mesoscopic sample is easily driven into the high-field regime; some tens of millivolts across a length of 100 nm will do it.[24] Nevertheless, although nonequilibrium noise is a unique source of dynamical information out of reach to linear-response theory,[25] none of the existing models has been pushed substantially beyond its low-field perturbative regime. The Landauer and Boltzmann-Langevin formalisms each suffers from its own obstacles to addressing strongly nonequilibrium effects, as does the quantum-kinetic theory of Altshuler, Levitov, and Yakovets,[26] which tries to unify them. There is, therefore, a real need for another approach: one that is nonperturbative in the driving field. This need has been reasserted very recently by

numerical evidence of a rich structure for shot noise at high fields, even in nondegenerate systems.[27]

We propose a direct and versatile method for treating noise semiclassically from equilibrium up to high fields, building on Fermi-liquid theory[2,28] and a family of Green functions for the linearised transport equation. Such functions have been studied by Kogan and Shul'man,[19] Gantsevich, Gurevich, and Katilius,[29] and by Stanton and Wilkins, in great detail, for semiconductors.[25,30] Unlike Boltzmann-Langevin, this approach is not limited to the specifically Boltzmannian form of the collision integral.[30] For every collisional approximation that can be used in the one-particle transport equation, there is a systematic construction for the two-particle Boltzmann-Green functions (BGFs). These then generate both the steady-state and transient nonequilibrium fluctuations, respecting the conservation laws as well as the one-body collisional structure and bringing great flexibility to noise calculations. An application to the noise perfomance of heterojunction transistors is given by Green and Chivers.[31]

In Section II we present a general framework for noise in small metallic systems, including Coulomb effects[27,32–34] where we identify two distinct mechanisms for screening. Sec. III contains the major applications of our theory. First, we extend the fluctuation-dissipation relation for thermally driven noise to nonequilibrium conductors,[35–37] highlighting the role of Pauli exclusion in suppressing hot-electron effects in the bulk noise spectrum.[1] We then discuss the many-body origin of shot noise and argue that, in their sharply contrasting responses to Coulomb screening, shot noise and thermal noise display quite different physics. We propose a simple experimental test of this difference. In Sec. IV we make an exact computation of shot noise within the Drude picture of a conducting wire. We include the effects of finite wire thickness on carrier motion, a significant source of shot-noise suppression at high currents that is unrelated to diffusive elastic scattering and to Coulomb screening. We sum up in Sec. V.

II. THEORY

The theoretical discussion is in four parts. We begin by formulating the transport problem as a direct mapping of the electron Fermi liquid to its nonequilibrium steady state. Second, we describe the steady-state fluctuations. Third, we discuss the dynamic fluctuations and their formal connection with the steady state. Last, we incorporate Coulomb screening into the nonequilibrium structure. Our end product is a complete expression for the current autocorrelation, which determines the noise.

A. Transport Model

The semiclassical Boltzmann transport equation (BTE) for the electron distribution function $f_\alpha(t) \equiv f_s(\mathbf{r}, \mathbf{k}, t)$ is

$$\left[\frac{\partial}{\partial t} + \mathbf{v}_{\mathbf{k}s}\cdot\frac{\partial}{\partial \mathbf{r}} - \frac{e\mathbf{E}(\mathbf{r},t)}{\hbar}\cdot\frac{\partial}{\partial \mathbf{k}}\right] f_\alpha(t) = -\sum_{\alpha'} \Big[W_{\alpha'\alpha}(1-f_{\alpha'})f_\alpha - W_{\alpha\alpha'}(1-f_\alpha)f_{\alpha'}\Big]. \tag{1}$$

Label $\alpha = \{\mathbf{r}, \mathbf{k}, s\}$ denotes a point in single-particle phase space, while sub-label s indexes both the discrete subbands (or valleys) of a multi-level system and the spin state. The system is acted upon by the total internal field $\mathbf{E}(\mathbf{r}, t)$. We study single-particle scattering, with a rate $W_{\alpha\alpha'} \equiv \delta(\mathbf{r}-\mathbf{r}')W_{ss'}(\mathbf{r}, \mathbf{k}, \mathbf{k}')$ that is local in real space, independent of the driving field, and that satisfies detailed balance: $W_{\alpha'\alpha}(1 - f^{\rm eq}_{\alpha'})f^{\rm eq}_\alpha = W_{\alpha\alpha'}(1 - f^{\rm eq}_\alpha)f^{\rm eq}_{\alpha'}$ where $f^{\rm eq}_\alpha$ is the equilibrium distribution. In a system with ν dimensions, we make the following correspondence for the identity operator:

$$\delta_{\alpha\alpha'} \equiv \delta_{ss'}\left\{\frac{\delta_{\mathbf{r}\mathbf{r}'}}{\Omega(\mathbf{r})}\right\}\{\Omega(\mathbf{r})\delta_{\mathbf{k}\mathbf{k}'}\} \longleftrightarrow \delta_{ss'}\delta(\mathbf{r}-\mathbf{r}')(2\pi)^\nu\delta(\mathbf{k}-\mathbf{k}').$$

The volume $\Omega(\mathbf{r})$ of a local cell in real space becomes the measure for spatial integration, while its inverse defines the scaling in wave-vector space for the local bands $\{\mathbf{k}, s\}$.

The first step is to construct the steady-state solution $f_\alpha \equiv f_\alpha(t \to \infty)$ explicitly from $f^{\rm eq}$, which satisfies the equilibrium, collisionless form of Eq. (1):

$$\mathbf{v}_{\mathbf{k}s}\cdot\frac{\partial f^{\rm eq}_\alpha}{\partial \mathbf{r}} - \frac{e\mathbf{E}_0(\mathbf{r})}{\hbar}\cdot\frac{\partial f^{\rm eq}_\alpha}{\partial \mathbf{k}} = 0. \tag{2}$$

The internal field $\mathbf{E}_0(\mathbf{r})$ is defined in the absence of a driving field. The quantities $f^{\rm eq}$ and $\mathbf{E}_0$ are linked self-consistently by the usual constitutive relations, the first being the Poisson equation

$$\frac{\partial}{\partial \mathbf{r}}\cdot\epsilon\mathbf{E}_0 = -4\pi e\Big(\langle f^{\rm eq}(\mathbf{r})\rangle - n^+(\mathbf{r})\Big) \tag{3}$$

in terms of the dielectric constant $\epsilon(\mathbf{r})$, the electron density $\langle f^{\rm eq}(\mathbf{r})\rangle \equiv \Omega(\mathbf{r})^{-1}\sum_{\mathbf{k},s} f^{\rm eq}_\alpha$, and the positive background density $n^+(\mathbf{r})$, which will remain unchanged throughout our calculations.[38] Normalisation to the total particle number is $\sum_{\mathbf{r}} \Omega(\mathbf{r})\langle f^{\rm eq}(\mathbf{r})\rangle = N$. The second relation is the form of the equilibrium function itself,

$$f^{\rm eq}_\alpha \;=\; \left[1+\exp\left(\frac{\varepsilon_\alpha - \phi_\alpha}{k_B T}\right)\right]^{-1}, \tag{4}$$

in which the local conduction-band energy $\varepsilon_\alpha = \varepsilon_s(\mathbf{k};\mathbf{r})$ can have band parameters that depend on position. The locally defined Fermi level $\phi_\alpha = \mu - V_0(\mathbf{r})$ is the difference of the global chemical potential μ and the electrostatic potential $V_0(\mathbf{r})$, whose gradient is $e\mathbf{E}_0(\mathbf{r})$.

Define the difference function $g_\alpha = f_\alpha - f^{\rm eq}_\alpha$. From each side of Eq. (1) in the steady state, subtract its equilibrium counterpart. We obtain

$$\begin{aligned}\mathbf{v}_{\mathbf{k}s}\cdot\frac{\partial g_\alpha}{\partial \mathbf{r}} - \frac{e\mathbf{E}(\mathbf{r})}{\hbar}\cdot\frac{\partial g_\alpha}{\partial \mathbf{k}} &= \frac{e(\mathbf{E}-\mathbf{E}_0)}{\hbar}\cdot\frac{\partial f^{\rm eq}_\alpha}{\partial \mathbf{k}} - \sum_{\alpha'}(W_{\alpha'\alpha} g_\alpha - W_{\alpha\alpha'} g_{\alpha'}) \\ &\quad + \sum_{\alpha'}(W_{\alpha'\alpha} - W_{\alpha\alpha'})(f^{\rm eq}_{\alpha'} g_\alpha + g_{\alpha'} f^{\rm eq}_\alpha + g_\alpha g_{\alpha'}).\end{aligned} \tag{5}$$

The solutions to Eqs. (2) and (5) are determined by the asymptotic conditions in the source and drain reservoirs, be it at equilibrium or with an external electromotive force. Our active region includes the carriers within the source and drain terminals out to several screening lengths, so that local fields are negligible at the interfaces with the reservoirs. In practice we assume that all fields are shorted out so that $\mathbf{E}(\mathbf{r}) = \mathbf{E}_0(\mathbf{r}) = \mathbf{0}$ beyond the boundaries. Gauss's theorem implies that this bounded system remains neutral overall:

$$\sum_{\mathbf{r}} \Omega(\mathbf{r})\langle g(\mathbf{r})\rangle = \sum_\alpha g_\alpha = 0. \tag{6}$$

We put Eq. (5) into integro-differential form, with an inhomogeneous term explicitly dependent on $f^{\rm eq}$:

$$\sum_{\alpha'} B[W^A f]_{\alpha\alpha'} g_{\alpha'} = \frac{e\widetilde{\mathbf{E}}(\mathbf{r})}{\hbar}\cdot\frac{\partial f^{\rm eq}_\alpha}{\partial \mathbf{k}} + \sum_{\alpha'} W^A_{\alpha\alpha'} g_{\alpha'} g_\alpha \tag{7}$$

where $\widetilde{\mathbf{E}}(\mathbf{r}) = \mathbf{E}(\mathbf{r}) - \mathbf{E}_0(\mathbf{r})$ is the local field induced by the external electromotive potential and $B[W^A f]$ is the linearised Boltzmann operator

$$\begin{aligned}B[W^A f]_{\alpha\alpha'} \stackrel{\rm def}{=} \delta_{\alpha\alpha'} &\left[\mathbf{v}_{\mathbf{k}'s'}\cdot\frac{\partial}{\partial \mathbf{r}'} - \frac{e\mathbf{E}(\mathbf{r}')}{\hbar}\cdot\frac{\partial}{\partial \mathbf{k}'} + \sum_\beta (W_{\beta\alpha'} - W^A_{\beta\alpha'} f_\beta)\right] \\ &- W_{\alpha\alpha'} + W^A_{\alpha\alpha'} f_\alpha,\end{aligned} \tag{8}$$

and $W^A_{\alpha\alpha'} = W_{\alpha\alpha'} - W_{\alpha'\alpha}$. Note that $W^A = 0$ if the scattering is elastic. To represent the physical solution, g must vanish with $\widetilde{E}$ in the equilibrium limit. This is guaranteed by the Poisson equation

$$\frac{\partial}{\partial \mathbf{r}}\cdot\epsilon\widetilde{\mathbf{E}} = -4\pi e\Big(\langle f(\mathbf{r})\rangle - \langle f^{\rm eq}(\mathbf{r})\rangle\Big) = -4\pi e\langle g(\mathbf{r})\rangle. \tag{9}$$

B. Boltzmann-Green Functions

To calculate the adiabatic response of the system about its nonequilibrium steady state we introduce the Boltzmann-Green function[39]

$$G_{\alpha\alpha'} \stackrel{\rm def}{=} \frac{\delta g_\alpha}{\delta f^{\rm eq}_{\alpha'}}, \tag{10}$$

with a global constraint following directly from Eq. (6):

$$\sum_{\alpha} G_{\alpha\alpha'} = 0. \tag{11}$$

The equation of motion for G is derived by taking variations on both sides of Eq. (5):

$$\sum_{\beta} B[W^A f]_{\alpha\beta} G_{\beta\alpha'} = \delta_{\alpha\alpha'} \left[\frac{e\widetilde{\mathbf{E}}(\mathbf{r}')}{\hbar} \cdot \frac{\partial}{\partial \mathbf{k}'} + \sum_{\beta} W^A_{\beta\alpha'} g_\beta \right] - W^A_{\alpha\alpha'} g_\alpha. \tag{12}$$

The variation is restricted by excluding the reaction of the local fields $\mathbf{E}_0(\mathbf{r})$ and $\widetilde{\mathbf{E}}(\mathbf{r})$. This means that G is a response function free of Coulomb screening. Here we treat the electrons as an effectively neutral Fermi liquid; in Section IID we give the complete fluctuation structure, with Coulomb effects.

All of the steady-state fluctuation properties induced by the thermal background are specified in terms of G and the equilibrium fluctuation Δf^{eq}. The equilibrium fluctuation is the proper particle-particle correlation in the static long-wavelength limit, normalised by the thermal energy $k_B T$. In the Lindhard approximation[2] this is

$$\Delta f^{\mathrm{eq}}_\alpha \equiv k_B T \frac{\partial f^{\mathrm{eq}}_\alpha}{\partial \phi_\alpha} = f^{\mathrm{eq}}_\alpha (1 - f^{\mathrm{eq}}_\alpha). \tag{13}$$

When there are strong exchange-correlation interactions, this two-body expression is renormalised by a factor dependent on the Landau quasiparticle parameters.[2,28] In this work we consider free electrons only.

Define the two-particle fluctuation function $\Delta f^{(2)}_{\alpha\alpha'} \equiv (\delta_{\alpha\alpha'} + G_{\alpha\alpha'}) \Delta f^{\mathrm{eq}}_{\alpha'}$. The steady-state distribution of the particle-number fluctuation is the sum of all of the two-body terms:

$$\Delta f_\alpha = \sum_{\alpha'} \Delta f^{(2)}_{\alpha\alpha'} = \Delta f^{\mathrm{eq}}_\alpha + \sum_{\alpha'} G_{\alpha\alpha'} \Delta f^{\mathrm{eq}}_{\alpha'}. \tag{14}$$

The nonequilibrium fluctuation Δf, manifestly a linear functional of its equilibrium Fermi-Dirac form, is the exact solution to the linearised Boltzmann equation:

$$\sum_{\beta} B[W^A f]_{\alpha\beta} \Delta f_\beta = 0. \tag{15}$$

Charge neutrality implies that the total fluctuation strength over the sample, $\Delta N = \sum_{\mathbf{r}} \Omega(\mathbf{r}) \langle \Delta f(\mathbf{r}) \rangle$, is conserved. This constrains both steady-state and dynamical fluctuations.

Calculation of the dynamic response requires the time-dependent Boltzmann-Green function[19]

$$R_{\alpha\alpha'}(t - t') \stackrel{\mathrm{def}}{=} \theta(t - t') \frac{\delta f_\alpha(t)}{\delta f_{\alpha'}(t')}, \tag{16}$$

with initial value $R_{\alpha\alpha'}(0) = \delta_{\alpha\alpha'}$. As with G, the variation is restricted. The linearised BTE satisfied by $R(t - t')$ is derived from Eq. (1) and takes the form

$$\sum_{\beta} \left\{ \delta_{\alpha\beta} \frac{\partial}{\partial t} + B[W^A f]_{\alpha\beta} \right\} R_{\beta\alpha'}(t - t') = \delta(t - t') \delta_{\alpha\alpha'}. \tag{17}$$

Summation over α on both sides of this equation leads to conservation of normalisation:[19]

$$\sum_{\alpha} R_{\alpha\alpha'}(t - t') = \theta(t - t'). \tag{18}$$

The time-dependent BGF is a two-point correlation. It tracks the history of a free electron in state α' added to the system at time t'; the probability of finding the electron in state α at time t is just $R_{\alpha\alpha'}(t - t')$. In the long-time limit Eq. (17) goes to its steady-state form, and so $R_{\alpha\alpha'}(t \to \infty) \propto \Delta f_\alpha$, the solution to the steady-state linearised equation.[19] Together with Eq. (18) this gives the identity

$$R_{\alpha\alpha'}(t \to \infty) = \frac{\Delta f_\alpha}{\Delta N}. \tag{19}$$

All of the time-dependent fluctuation properties induced by the thermal background are specified in terms of R and the steady-state nonequilibrium fluctuation Δf. From the dynamical two-particle distribution,[29] that is $\Delta f^{(2)}_{\alpha\alpha'}(t) \equiv R_{\alpha\alpha'}(t)\Delta f_{\alpha'}$, one can construct the lowest-order moment

$$\Delta f_{\alpha}(t) = \sum_{\alpha'} \Delta f^{(2)}_{\alpha\alpha'}(t) \tag{20}$$

in analogy with Eq. (14). Equation (17) has an adjoint,[19] with whose help one can show that $\Delta f_{\alpha}(t) = \Delta f_{\alpha}$ for $t > 0$. Thus the intrinsic time dependence of $\Delta f^{(2)}(t)$ is not revealed through this quantity. [Note: a remark in Green, Ref. 39, that $\Delta f(t)$ is inherently time-dependent, is true only for collision-time approximations.] Eq. (18) implies that the total fluctuation strength is constant: $\sum_{\mathbf{r}} \Omega(\mathbf{r}) \langle \Delta f(\mathbf{r}, t) \rangle = \Delta N$.

C. Dynamic Correlations

We move to the frequency domain. An important outcome of this analysis is the extension of the fluctuation-dissipation (FD) relation to the nonequilibrium regime. This requires expressing both the difference function g and the adiabatic Boltzmann-Green function G in terms of the dynamical response. The Fourier transform $\mathrm{R}(\omega)$ of the time-dependent BGF satisfies

$$\sum_{\beta} \big\{ B[W^A f]_{\alpha\beta} - i\omega\delta_{\alpha\beta} \big\} \mathrm{R}_{\beta\alpha'}(\omega) = \delta_{\alpha\alpha'}, \tag{21}$$

making $\mathrm{R}(\omega)$ the resolvent for the linearised operator of Eq. (8). The global condition on $\mathrm{R}(\omega)$ from Eq. (18) is

$$\sum_{\alpha} \mathrm{R}_{\alpha\alpha'}(\omega) = -\frac{1}{i(\omega + i\eta)}, \qquad \eta \to 0^{+}. \tag{22}$$

At first sight this fails to match the corresponding condition on the adiabatic Boltzmann-Green function, Eq. (11). To determine the solution of Eq. (12) for G in terms of the resolvent, we follow Kogan and Shul'man[19] and introduce the intrinsically correlated part of $\mathrm{R}(\omega)$, namely

$$\mathrm{C}_{\alpha\alpha'}(\omega) = \mathrm{R}_{\alpha\alpha'}(\omega) + \frac{1}{i(\omega + i\eta)} \frac{\Delta f_{\alpha}}{\Delta N}. \tag{23}$$

This correlated propagator satisfies a pair of identities in the frequency domain;[19] the transform of the relation $\Delta f(t) = \theta(t)\Delta f$ leads to

$$\sum_{\alpha'} \mathrm{C}_{\alpha\alpha'}(\omega) \Delta f_{\alpha'} = 0 \tag{24a}$$

while Eq. (22) leads to

$$\sum_{\alpha} \mathrm{C}_{\alpha\alpha'}(\omega) = 0. \tag{24b}$$

The second of these corresponds to the constraint on G. Like $\mathrm{R}(\omega)$, the correlated BGF is analytic in the upper half-plane $\mathrm{Im}\{\omega\} > 0$, and satisfies the Kramers-Krönig dispersion relations. Unlike $\mathrm{R}(\omega)$, however, $\mathrm{C}(\omega)$ is regular for $\omega \to 0$.

We now obtain g and G in terms of the correlated dynamical propagator. Consider the equation

$$\sum_{\alpha'} \big\{ B[W^A f]_{\alpha\alpha'} - i\omega\delta_{\alpha\alpha'} \big\} \mathrm{g}_{\alpha'}(\omega) = \frac{e\widetilde{\mathbf{E}}(\mathbf{r})}{\hbar} \cdot \frac{\partial f^{\mathrm{eq}}_{\alpha}}{\partial \mathbf{k}} + \sum_{\alpha'} g_{\alpha} W^A_{\alpha\alpha'} g_{\alpha'}, \tag{25}$$

with solution

$$\mathrm{g}_{\alpha}(\omega) = \sum_{\alpha'} \mathrm{C}_{\alpha\alpha'}(\omega) \frac{e\widetilde{\mathbf{E}}(\mathbf{r}')}{\hbar} \cdot \frac{\partial f^{\mathrm{eq}}_{\alpha'}}{\partial \mathbf{k}'} + \sum_{\alpha'\beta} \mathrm{C}_{\alpha\alpha'}(\omega) g_{\alpha'} W^A_{\alpha'\beta} g_{\beta}. \tag{26}$$

The uncorrelated component of $\mathrm{R}(\omega)$ does not contribute to the right-hand side of this equation; in the first term it results in a decoupling of the summation over α', yielding zero because $\partial f^{\rm eq}_{\alpha'}/\partial\mathbf{k}'$ is odd in $\mathbf{k}'$. In the second term, decoupling means that the double summation over α' and β vanishes by antisymmetry. In the static limit Eq. (25) becomes the inhomogeneous equation (7), and moreover $\mathrm{g}(0)$ satisfies Eq. (6), the sum rule for g. Therefore

$$g_\alpha = \sum_{\alpha'} \mathrm{C}_{\alpha\alpha'}(0) \frac{e\widetilde{\mathbf{E}}(\mathbf{r}')}{\hbar} \cdot \frac{\partial f^{\rm eq}_{\alpha'}}{\partial \mathbf{k}'} + \sum_{\alpha'\beta} \mathrm{C}_{\alpha\alpha'}(0) g_{\alpha'} W^A_{\alpha'\beta} g_\beta. \tag{27}$$

This identity is central to the FD relation.

In models with symmetric scattering W^A is zero and the adiabatic BGF assumes a simple form on varying both sides of Eq. (27):

$$G_{\alpha\alpha'} = \mathrm{C}_{\alpha\alpha'}(0) \frac{e\widetilde{\mathbf{E}}(\mathbf{r}')}{\hbar} \cdot \frac{\partial}{\partial \mathbf{k}'}. \tag{28}$$

More generally, an analysis similar to that for $\mathrm{g}(\omega)$ can be used directly for the adiabatic propagator. Introduce the operator $\mathrm{G}(\omega)$, defined to satisfy the dynamic extension of Eq. (12),

$$\sum_\beta \{B[W^A f]_{\alpha\beta} - i\omega\delta_{\alpha\beta}\} \mathrm{G}_{\beta\alpha'}(\omega) = \delta_{\alpha\alpha'} \left[\frac{e\widetilde{\mathbf{E}}(\mathbf{r}')}{\hbar} \cdot \frac{\partial}{\partial \mathbf{k}'} + \sum_\beta W^A_{\beta\alpha'} g_\beta \right] - W^A_{\alpha\alpha'} g_\alpha. \tag{29}$$

This has the solution

$$\mathrm{G}_{\alpha\alpha'}(\omega) = \mathrm{C}_{\alpha\alpha'}(\omega) \frac{e\widetilde{\mathbf{E}}(\mathbf{r}')}{\hbar} \cdot \frac{\partial}{\partial \mathbf{k}'} - \sum_\beta \Big(\mathrm{C}_{\alpha\alpha'}(\omega) - \mathrm{C}_{\alpha\beta}(\omega) \Big) W^A_{\alpha'\beta} g_\beta. \tag{30}$$

In the first term on the right-hand side, the uncorrelated component of $\mathrm{R}(\omega)$ makes no contribution after decoupling because the physical distributions F_α on which $\mathrm{G}(\omega)$ operates vanish sufficiently fast that $\sum_{\mathbf{k}} \partial F_\alpha/\partial\mathbf{k} = \mathbf{0}$. In the second right-hand term the uncorrelated parts of $\mathrm{R}_{\alpha\alpha'}$ and $\mathrm{R}_{\alpha\beta}$ cancel directly. We conclude as before that

$$G_{\alpha\alpha'} = \mathrm{C}_{\alpha\alpha'}(0) \frac{e\widetilde{\mathbf{E}}(\mathbf{r}')}{\hbar} \cdot \frac{\partial}{\partial \mathbf{k}'} - \sum_\beta \Big(\mathrm{C}_{\alpha\alpha'}(0) - \mathrm{C}_{\alpha\beta}(0) \Big) W^A_{\alpha'\beta} g_\beta. \tag{31}$$

This is a crucial result because it shows (a) that the adiabatic structure of the steady state, through G, is of one piece with the dynamics, and (b) that the nonequilibrium correlations originate *manifestly* from the equilibrium state, through $G\Delta f^{\rm eq}$. We have thus proved that the kinetic BGF analysis is formally self-sufficient once its boundary conditions are given. This fact is embodied in the frequency sum rule

$$\frac{\delta g_\alpha}{\delta f^{\rm eq}_{\alpha'}} \;=\; \frac{2}{\pi} \int_0^\infty \frac{d\omega}{\omega} \mathrm{Im}\{\mathrm{G}_{\alpha\alpha'}(\omega)\}. \tag{32}$$

The vehicle for the physics of current noise is the velocity autocorrelation. It is a two-point distribution in real space, built on the correlated part of the two-particle fluctuation $\Delta\mathrm{f}^{(2)}_{\alpha\alpha'} = \mathrm{R}_{\alpha\alpha'}\Delta f_{\alpha'}$ and taking the form[29]

$$\langle\!\langle \mathbf{v}\mathbf{v}'\Delta\mathrm{f}^{(2)}(\mathbf{r},\mathbf{r}';\omega)\rangle\!\rangle'_c \stackrel{\rm def}{=} \frac{1}{\Omega(\mathbf{r})} \frac{1}{\Omega(\mathbf{r}')} \sum_{\mathbf{k},s} \sum_{\mathbf{k}',s'} \mathbf{v}_{\mathbf{k}s} \mathrm{Re}\{\mathrm{C}_{\alpha\alpha'}(\omega)\} \mathbf{v}_{\mathbf{k}'s'} \Delta f_{\alpha'}. \tag{33}$$

The nonlocal velocity autocorrelation provides the direct basis for shot-noise calculations when the distance $|\mathbf{r}-\mathbf{r}'|$ becomes comparable to the mean free path. The local function derived from it,

$$S_f(\mathbf{r},\omega) = e^2 \sum_{\mathbf{r}'} \Omega(\mathbf{r}') \langle\!\langle (\widetilde{\mathbf{E}}(\mathbf{r})\cdot\mathbf{v})(\widetilde{\mathbf{E}}(\mathbf{r}')\cdot\mathbf{v}')\Delta\mathrm{f}^{(2)}(\mathbf{r},\mathbf{r}';\omega)\rangle\!\rangle'_c, \tag{34}$$

has a macroscopic reach since in effect it samples fluctuations over the bulk. It is closely related to the current-noise spectral density and satisfies the nonequilibrium FD relation discussed in Sec. III.

D. Coulomb Effects

We generate the Boltzmann-Green functions in the presence of induced fluctuations of the electric fields. Variations are now unrestricted. The resulting description of screening effects extends the physics of classical space-charge suppression.[40] We consider samples with a fixed dielectric constant, and likewise for the reservoirs.

The equation for the screened equilibrium fluctuation $\widetilde{\Delta} f^{\rm eq} \equiv k_B T \delta f^{\rm eq}/\delta\mu$ is obtained by operating on Eq. (2) satisfied by the equilibrium distribution. We have

$$\mathbf{v}_{\mathbf{k}s}\cdot\frac{\partial \widetilde{\Delta} f^{\rm eq}_{\alpha}}{\partial \mathbf{r}} - \frac{e\mathbf{E}_0(\mathbf{r})}{\hbar}\cdot\frac{\partial \widetilde{\Delta} f^{\rm eq}_{\alpha}}{\partial \mathbf{k}} = \left(\frac{e}{\hbar}\sum_{\alpha'}\frac{\delta \mathbf{E}_0(\mathbf{r})}{\delta f^{\rm eq}_{\alpha'}}\widetilde{\Delta} f^{\rm eq}_{\alpha'}\right)\cdot\frac{\partial f^{\rm eq}_{\alpha}}{\partial \mathbf{k}}. \tag{35}$$

Detailed balance keeps the equation collisionless while the Poisson equation (3) implies that, within the system boundaries, the variation of $e\mathbf{E}_0$ with respect to $f^{\rm eq}$ is the Coulomb force for an electron,

$$e\frac{\delta \mathbf{E}_0(\mathbf{r})}{\delta f^{\rm eq}_{\alpha'}} = -e\mathbf{E}_C(\mathbf{r}-\mathbf{r}') \equiv \frac{\partial}{\partial \mathbf{r}}V_C(\mathbf{r}-\mathbf{r}') \tag{36}$$

where $V_C(\mathbf{r}) = e^2/\epsilon|\mathbf{r}|$ is the Coulomb potential. As a result Eq. (35) becomes

$$\mathbf{v}_{\mathbf{k}s}\cdot\frac{\partial \widetilde{\Delta} f^{\rm eq}_{\alpha}}{\partial \mathbf{r}} - \frac{e\mathbf{E}_0(\mathbf{r})}{\hbar}\cdot\frac{\partial \widetilde{\Delta} f^{\rm eq}_{\alpha}}{\partial \mathbf{k}} = -\frac{e}{\hbar}\sum_{\alpha'}\frac{\partial f^{\rm eq}_{\alpha}}{\partial \mathbf{k}}\cdot\mathbf{E}_C(\mathbf{r}-\mathbf{r}')\widetilde{\Delta} f^{\rm eq}_{\alpha'}. \tag{35$'$}$$

Viewed as a variant of the equilibrium BTE, Eq. (35′) is inhomogeneous. Its solution includes a term proportional to the homogeneous solution, which in this case is the bare fluctuation $\Delta f^{\rm eq}$. Let γ_C be the proportionality constant. Then

$$\widetilde{\Delta} f^{\rm eq}_{\alpha} = \gamma_C \Delta f^{\rm eq}_{\alpha} - \frac{e}{\hbar}\sum_{\alpha'\alpha''} {\rm C}^{\rm eq}_{\alpha\alpha'}(0)\frac{\partial f^{\rm eq}_{\alpha'}}{\partial \mathbf{k}'}\cdot\mathbf{E}_C(\mathbf{r}'-\mathbf{r}'')\widetilde{\Delta} f^{\rm eq}_{\alpha''}, \tag{37}$$

in which ${\rm C}^{\rm eq}$ is the correlated part of the resolvent for the equilibrium state. The integral on the right-hand side of Eq. (37) has a structure similar to Eq. (27), in that the uncorrelated part of the resolvent gives no contribution after decoupling of the intermediate wave-vector sums.

1. Thomas-Fermi Screening

The constant γ_C is sensitive to the physics of charge transfer between sample and reservoirs. Recall that the fluctuation $\Delta f^{\rm eq}_{\alpha} = k_B T \partial f^{\rm eq}_{\alpha}/\partial \phi_{\alpha}$ is a measure of the electrons' response, as a Fermi liquid, to a change in the effective Fermi level $\phi_{\alpha} = \mu - V_0(\mathbf{r})$. The latter is the net contribution from kinematics *alone* to the cost of adding an electron locally to the system; the electrostatic energy $V_0(\mathbf{r})$ is excluded from the Fermi-liquid accounting.

When the Coulomb fields are frozen, as in the restricted analysis, the Fermi-level variation is that of the global chemical potential: $\delta\phi_{\alpha} = \delta\mu$. In the full Coulomb problem, we must offset the energy cost of charge transfer from reservoir to sample. The Coulomb energy needed to add an electron to the conductor is

$$u_c = \frac{1}{N}\sum_{\alpha} V_0(\mathbf{r}) f^{\rm eq}_{\alpha}. \tag{38}$$

A corresponding term u_r characterises the reservoirs. However, u_r cannot be probed directly; its effects are absorbed within the operational definition of the chemical potential. This means that $u_r = 0$ identically, and that u_c thus represents the net work to move an electron from reservoir to sample. It is the conduction-electron contribution to the contact potential.[3] (By contrast, the core-electron contribution determines the offset in the band bottom $\varepsilon_s(\mathbf{k}{=}\mathbf{0};\mathbf{r})$. This is independent of μ and does not appear explicitly in the variational analysis.)

It follows that the portion of the chemical potential sustaining the Fermi liquid in the conductor is $\mu - u_c$. Free variation of the global parameter μ generates the coefficient

$$\gamma_C = \frac{\delta}{\delta\mu}(\mu - u_c) = 1 - \frac{\delta u_c}{\delta\mu}. \tag{39}$$

Application to Eq. (37) of Eq. (24b), that is the sum rule $\sum_\alpha \mathrm{C}_{\alpha\alpha'} = 0$, establishes the normalisation

$$\sum_\alpha \widetilde{\Delta} f^{\mathrm{eq}}_\alpha = \gamma_C \Delta N. \tag{40}$$

If sample and reservoir have matching electronic properties, then $u_c = u_r \equiv 0$ and $\gamma_C = 1$. This is the norm for noise measurements in metallic wires. If, on the other hand, the sample differs substantially from the reservoir in metallic structure, then from Eq. (38) and the form for $V_0(\mathbf{r})$ as the solution to the Poisson equation (3) one obtains

$$\gamma_C = 1 - \frac{1}{N}\sum_\alpha \left(\sum_{\alpha'} V_C(\mathbf{r}-\mathbf{r}') f^{\mathrm{eq}}_{\alpha'} + V_0(\mathbf{r}) - \frac{u_c}{N} \right) \frac{\widetilde{\Delta} f^{\mathrm{eq}}_\alpha}{k_B T}, \tag{41}$$

an instance of suppression by self-consistent Thomas-Fermi screening.

There is strong indirect evidence that this mechanism is the major determinant of low-noise performance in heterojunction field-effect devices.[31] At mesoscopic scales, one can anticipate a wide variety of interfacial screening behaviours for the fluctuations. Potentially interesting is the case of metal–semiconductor–metal structures. See Appendix A, Eq. (A10).

Note that γ_C enters only at the two-body level. It cannot renormalise the one-body distribution g, or any averages constructed with g. This includes transport coefficients such as the mobility.

Equation (37) is solved by introducing a Coulomb screening operator $\Gamma^{\mathrm{eq}}(0)$, whose inverse is

$$\Big(\Gamma^{\mathrm{eq}}(0)^{-1}\Big)_{\alpha\alpha'} = \delta_{\alpha\alpha'} + \frac{e}{\hbar}\sum_\beta \mathrm{C}^{\mathrm{eq}}_{\alpha\beta}(0) \frac{\partial f^{\mathrm{eq}}_\beta}{\partial \mathbf{k}_\beta} \cdot \mathbf{E}_C(\mathbf{r}_\beta - \mathbf{r}'). \tag{42a}$$

This yields

$$\widetilde{\Delta} f^{\mathrm{eq}}_\alpha = \sum_{\alpha'} \Gamma^{\mathrm{eq}}_{\alpha\alpha'}(0) \Big(\gamma_C \Delta f^{\mathrm{eq}}_{\alpha'} \Big), \tag{42b}$$

analogous to the Lindhard screening theory of the electron gas in the static limit.[2] Taken together with Eq. (41), it allows a closed-form solution for γ_C.

2. Collision-Mediated Screening

Away from equilibrium we define the Coulomb screening operator through its inverse

$$\Big(\Gamma(\omega)^{-1}\Big)_{\alpha\alpha'} \stackrel{\mathrm{def}}{=} \delta_{\alpha\alpha'} + \frac{e}{\hbar}\sum_\beta \mathrm{C}_{\alpha\beta}(\omega) \frac{\partial f_\beta}{\partial \mathbf{k}_\beta} \cdot \mathbf{E}_C(\mathbf{r}_\beta - \mathbf{r}'). \tag{43}$$

While γ_C is collisionless, the operator Γ captures the dynamics of interaction between scattering and screening, an exclusively nonequilibrium process. A few elementary results for both collisional and Thomas-Fermi screening are discussed in Appendix A. An important property of the collision-mediated screening operator, following from Eq. (24b), is

$$\sum_\alpha \Gamma_{\alpha\alpha'}(\omega) = 1. \tag{44}$$

The significance of Γ first becomes evident in obtaining the screened adiabatic propagator $\widetilde{G}_{\alpha\alpha'} = \delta g_\alpha / \delta f^{\mathrm{eq}}_{\alpha'}$, whose unrestricted BTE [cf Eq. (12) for G] is

$$\begin{aligned} \sum_\beta B[W^A f]_{\alpha\beta} \widetilde{G}_{\beta\alpha'} = \sum_\beta B[W^A f]_{\alpha\beta} G_{\beta\alpha'} + \frac{e}{\hbar} \left(\sum_\beta \frac{\delta \mathbf{E}(\mathbf{r})}{\delta f_\beta} \frac{\delta f_\beta}{\delta f^{\mathrm{eq}}_{\alpha'}} \right) \cdot \frac{\partial f_\alpha}{\partial \mathbf{k}} \\ - \frac{e}{\hbar} \frac{\delta \mathbf{E}_0(\mathbf{r})}{\delta f^{\mathrm{eq}}_{\alpha'}} \cdot \frac{\partial f^{\mathrm{eq}}_\alpha}{\partial \mathbf{k}} \end{aligned} \tag{45}$$

Poisson's equation (9) once again determines the variation of $\mathbf{E}$ with respect to f as

$$\frac{\delta \mathbf{E}(\mathbf{r})}{\delta f_{\alpha'}} = -\mathbf{E}_C(\mathbf{r} - \mathbf{r}'). \tag{46}$$

The solution of Eq. (45) is a two-pass process, in which one first resolves the Boltzmann operator $B[W^A f]$ and then invokes Γ to rationalise the transformed equation:

$$\begin{aligned}\widetilde{G}_{\alpha\alpha'} &= G_{\alpha\alpha'} - \frac{e}{\hbar}\sum_{\beta\beta'} \mathrm{C}_{\alpha\beta}(0)\frac{\partial f_\beta}{\partial \mathbf{k}_\beta}\boldsymbol{\cdot}\mathbf{E}_C(\mathbf{r}_\beta - \mathbf{r}_{\beta'})\Big(\delta_{\beta'\alpha'} + \widetilde{G}_{\beta'\alpha'}\Big) \\ &\quad + \frac{e}{\hbar}\sum_{\beta} \mathrm{C}_{\alpha\beta}(0)\frac{\partial f^{\mathrm{eq}}_\beta}{\partial \mathbf{k}_\beta}\boldsymbol{\cdot}\mathbf{E}_C(\mathbf{r}_\beta - \mathbf{r}') \\ &= \sum_\beta \Gamma_{\alpha\beta}(0)\left(G_{\beta\alpha'} - \frac{e}{\hbar}\sum_{\beta'} \mathrm{C}_{\beta\beta'}(0)\frac{\partial g_{\beta'}}{\partial \mathbf{k}_{\beta'}}\boldsymbol{\cdot}\mathbf{E}_C(\mathbf{r}_{\beta'} - \mathbf{r}')\right).\end{aligned} \tag{47}$$

We have also used $\delta f_{\beta'}/\delta f^{\mathrm{eq}}_{\alpha'} = \delta_{\beta'\alpha'} + \widetilde{G}_{\beta'\alpha'}$. The screened steady-state fluctuation is now

$$\widetilde{\Delta} f_\alpha = \widetilde{\Delta} f^{\mathrm{eq}}_\alpha + \sum_{\alpha'} \widetilde{G}_{\alpha\alpha'}\widetilde{\Delta} f^{\mathrm{eq}}_{\alpha'}. \tag{48}$$

The main outcome of the structure of $\widetilde{G}$ is the invariance of the fluctuation strength over the sample; this follows from $\sum_\alpha \widetilde{G}_{\alpha\alpha'} = 0$. Thus

$$\sum_\alpha \widetilde{\Delta} f_\alpha = \gamma_C \Delta N \equiv \widetilde{\Delta} N. \tag{49}$$

3. Dynamics

We now examine the screened dynamics. In the time domain the screened resolvent $\widetilde{R}(t)$ has the same formal definition, Eq. (16), as its restricted analogue. The unrestricted equation of motion, Fourier transformed, is

$$\sum_\beta \big\{B[W^A f]_{\alpha\beta} - i\omega\delta_{\alpha\beta}\big\}\widetilde{\mathrm{R}}_{\beta\alpha'}(\omega) = \delta_{\alpha\alpha'} - \frac{e}{\hbar}\sum_\beta \frac{\partial f_\alpha}{\partial \mathbf{k}}\boldsymbol{\cdot}\mathbf{E}_C(\mathbf{r} - \mathbf{r}_\beta)\widetilde{\mathrm{R}}_{\beta\alpha'}(\omega), \tag{50}$$

with solution

$$\widetilde{\mathrm{R}}_{\alpha\alpha'}(\omega) = \sum_\beta \Gamma_{\alpha\beta}(\omega)\mathrm{R}_{\beta\alpha'}(\omega). \tag{51}$$

This resolvent obeys identities analogous to those for $\mathrm{R}(\omega)$, namely Eq. (22) and, in the time domain, $\widetilde{R}_{\alpha\alpha'}(t \to \infty) = \widetilde{\Delta} f_\alpha / \widetilde{\Delta} N$. Together with Eq. (49), equality of the residues at $\omega = 0$ on each side of Eq. (51) implies the relation

$$\widetilde{\Delta} f_\alpha = \gamma_C \sum_{\alpha'} \Gamma_{\alpha\alpha'}(0)\Delta f_{\alpha'}, \tag{52}$$

equivalent to Eq. (48) by the properties of G and $B[W^A f]$.

The correlated propagator, with screening, is

$$\widetilde{\mathrm{C}}_{\alpha\alpha'}(\omega) = \widetilde{\mathrm{R}}_{\alpha\alpha'}(\omega) + \frac{1}{i(\omega + i\eta)}\frac{\widetilde{\Delta} f_\alpha}{\widetilde{\Delta} N}. \tag{53}$$

Its structure follows from combining Eqs. (23), (51), and (52) for

$$\widetilde{\mathrm{C}}_{\alpha\alpha'}(\omega) = \sum_\beta \Gamma_{\alpha\beta}(\omega)\mathrm{C}_{\beta\alpha'}(\omega) - \gamma_C \sum_\beta \left(\frac{\Gamma_{\alpha\beta}(\omega) - \Gamma_{\alpha\beta}(0)}{i\omega}\right)\frac{\Delta f_\beta}{\widetilde{\Delta} N}. \tag{54}$$

Applying the screened form of Eq. (24a), that is $\sum_{\beta'} \widetilde{\mathrm{C}}_{\beta\beta'}(\omega)\widetilde{\Delta} f_{\beta'} = 0$, to Eq. (54) generates

$$\sum_{\beta\beta'} \Gamma_{\alpha\beta}(\omega)\mathrm{C}_{\beta\beta'}(\omega)\widetilde{\Delta} f_{\beta'} = \gamma_C \sum_{\beta} \left(\frac{\Gamma_{\alpha\beta}(\omega) - \Gamma_{\alpha\beta}(0)}{i\omega} \right) \Delta f_{\beta}.$$

Fed back into Eq. (54), this produces

$$\widetilde{\mathrm{C}}_{\alpha\alpha'}(\omega) = \sum_{\beta\beta'} \Gamma_{\alpha\beta}(\omega)\mathrm{C}_{\beta\beta'}(\omega)\left(\delta_{\beta'\alpha'} - \frac{\widetilde{\Delta} f_{\beta'}}{\widetilde{\Delta} N} \right). \tag{55}$$

4. Current-Current Correlation

All the components are in place to construct the velocity autocorrelation function in the presence of screening. At zero frequency this is

$$\langle\langle \mathbf{v}\mathbf{v}' \widetilde{\Delta}\mathrm{f}^{(2)}(\mathbf{r},\mathbf{r}';0) \rangle\rangle_c' \stackrel{\mathrm{def}}{=} \frac{1}{\Omega(\mathbf{r})}\frac{1}{\Omega(\mathbf{r}')} \sum_{\mathbf{k},s}\sum_{\mathbf{k}',s'} \mathbf{v}_{\mathbf{k}s} \widetilde{\mathrm{C}}_{\alpha\alpha'}(0) \mathbf{v}_{\mathbf{k}'s'} \widetilde{\Delta} f_{\alpha'}. \tag{56}$$

At finite frequency we must add the displacement-current contribution to the fluctuations. The velocity is replaced with the nonlocal operator

$$\mathbf{u}_{\mathbf{k}s}(\mathbf{r},\mathbf{r}'';\omega) \equiv \frac{\delta_{\mathbf{r}\mathbf{r}''}}{\Omega(\mathbf{r})}\mathbf{v}_{\mathbf{k}s} - \frac{i\omega\epsilon}{4\pi e}\mathbf{E}_C(\mathbf{r}-\mathbf{r}''), \tag{57}$$

which requires two intermediate spatial sums to be incorporated within the expectation $\langle\langle \mathrm{Re}\{\mathbf{u}\widetilde{\Delta}\mathrm{f}^{(2)}\mathbf{u}'^{*}\} \rangle\rangle_c'$. For $\omega = 0$ this recovers Eq. (56), a more complex expression than its bare counterpart Eq. (33). In practice, collisional Coulomb effects are dominant in mesoscopic and in strongly inhomogeneous systems.[27,33]

III. APPLICATIONS

The first of our applications connects thermal fluctuations and dissipation in the bulk nonequilibrium context. Little is known of the effects of degeneracy on noise beyond the linear limit,[1] and we analyse them here. In our second application we investigate the many-body nature of mesoscopic shot noise. For degenerate electrons we show that thermal and shot noise have very different physical properties not easily subsumed under a single formula.[11]

A. Nonequilibrium Fluctuation-Dissipation Relation

The fluctuation-dissipation relation near equilibrium connects the spectral density of the thermal fluctuations to the dissipative effects of the steady current in the system. However, dissipation by itself does not exhaust the physics of this sum rule. There are nonlinear terms, negligible in linear response, that dominate the high-field behaviour of the current noise.[25,31] We calculate these contributions. Since the relation is macroscopic, to lowest order we omit Coulomb screening effects; these are weak in the bulk metallic limit (see for example Appendix A).

The resolvent property of $\mathrm{R}(\omega)$ provides a formal connection between the steady-state (one-body) solution g and the dynamical (two-body) fluctuation $\Delta\mathrm{f}^{(2)}$ at the semiclassical level. Taken to its equilibrium limit this becomes the familiar theorem.[2] The connection is made in two steps. Consider the kinematic identity

$$\frac{\partial f^{\mathrm{eq}}_{\alpha}}{\partial \mathbf{k}} = -\frac{\hbar}{k_B T}\mathbf{v}_{\mathbf{k}s}\Delta f^{\mathrm{eq}}_{\alpha} \tag{58}$$

and apply it to the leading term on the right-hand side of Eq. (27). The result is

$$g_{\alpha} = -\frac{e}{k_B T}\sum_{\alpha'} \mathrm{C}_{\alpha\alpha'}(0)(\widetilde{\mathbf{E}}\cdot\mathbf{v})_{\alpha'}\Delta f^{\mathrm{eq}}_{\alpha'} + h_{\alpha}, \tag{59}$$

in which $h_\alpha = \sum_{\alpha'\beta} \mathrm{C}_{\alpha\alpha'}(0) g_{\alpha'} W^A_{\alpha'\beta} g_\beta$. Evaluating the current density according to $\mathbf{J}(\mathbf{r}) = -e\langle \mathbf{v} g\rangle$, the power density $P(\mathbf{r}) = \widetilde{\mathbf{E}}(\mathbf{r}){\cdot}\mathbf{J}(\mathbf{r})$ for Joule heating can be written as

$$P(\mathbf{r}) = \frac{e^2}{k_B T}\frac{1}{\Omega(\mathbf{r})}\sum_{\mathbf{k},s}\sum_{\alpha'}(\widetilde{\mathbf{E}}{\cdot}\mathbf{v})_\alpha \mathrm{C}_{\alpha\alpha'}(0)(\widetilde{\mathbf{E}}{\cdot}\mathbf{v})_{\alpha'}\Delta f^{\mathrm{eq}}_{\alpha'} - e\langle \widetilde{\mathbf{E}}{\cdot}\mathbf{v}h\rangle. \tag{60}$$

In the second step we take the one-point spectral function S_f in the static limit, substituting for Δf from Eq. (14) in the right-hand side of Eq. (34) to give

$$S_f(\mathbf{r},0) = \frac{e^2}{\Omega(\mathbf{r})}\sum_{\mathbf{k},s}\sum_{\mathbf{r}'}\sum_{\mathbf{k}',s'}(\widetilde{\mathbf{E}}{\cdot}\mathbf{v})_\alpha \mathrm{C}_{\alpha\alpha'}(0)(\widetilde{\mathbf{E}}{\cdot}\mathbf{v})_{\alpha'}\Delta f^{\mathrm{eq}}_{\alpha'} + S_g(\mathbf{r},0), \tag{61}$$

where $S_g(\mathbf{r},0)$ is generated by replacing Δf with $\Delta g = \sum G\Delta f^{\mathrm{eq}}$ in Eq. (33), and subsequently in Eq. (34). Direct comparison of Eqs. (60) and (61) leads to

$$\frac{S_f(\mathbf{r},0)}{k_B T} = P(\mathbf{r}) + e\langle \widetilde{\mathbf{E}}{\cdot}\mathbf{v}h\rangle + \frac{S_g(\mathbf{r},0)}{k_B T}. \tag{62}$$

This is the nonequilibrium FD relation.

The standard linear-response result follows. The term in h on the right-hand side varies as $\widetilde{E}g^2$, while the final term varies as $\widetilde{E}^2\Delta g$; therefore both of these contributions are of order $\widetilde{E}^3$. Suppose that the system is homogeneous and that $\widetilde{\mathbf{E}} = \mathbf{E}$ acts along the x-axis: then division by E^2 on both sides of Eq. (62) gives

$$\frac{1}{E^2}\frac{S_f(\mathbf{r},0)}{k_B T} \;\to\; \frac{|J_x|}{E} = \sigma, \tag{63}$$

where σ is the low-field conductivity. Eq. (63) is the near-equilibrium statement.

The purely nonequilibrium structures beyond $P(\mathbf{r})$ can be expanded similarly to it. We discuss the symmetric-scattering case, for which there is no contribution $e\langle \widetilde{\mathbf{E}}{\cdot}\mathbf{v}h\rangle$. Within S_g we apply the formula for the adiabatic Boltzmann-Green function, Eq. (28), to express Δg in terms of the correlated propagator C. This produces two equivalent closed forms for the higher-order correlation:

$$S_g(\mathbf{r},0) = \frac{e^2}{\Omega(\mathbf{r})}\sum_{\mathbf{k},s}\sum_{\beta}(\widetilde{\mathbf{E}}{\cdot}\mathbf{v})_\alpha \mathrm{C}_{\alpha\beta}(0)(\widetilde{\mathbf{E}}{\cdot}\mathbf{v})_\beta \sum_{\alpha'}\mathrm{C}_{\beta\alpha'}(0)\frac{e\widetilde{\mathbf{E}}(\mathbf{r}')}{\hbar}{\cdot}\frac{\partial\Delta f^{\mathrm{eq}}_{\alpha'}}{\partial\mathbf{k}'}, \tag{64a}$$

$$S_g(\mathbf{r},0) = -\frac{e^3}{k_B T}\frac{1}{\Omega(\mathbf{r})}\sum_{\mathbf{k},s}\sum_{\alpha'}(\widetilde{\mathbf{E}}{\cdot}\mathbf{v})_\alpha \left(\mathrm{C}(0)\widetilde{\mathbf{E}}{\cdot}\mathbf{v}\right)^2_{\alpha\alpha'}(1-2f^{\mathrm{eq}}_{\alpha'})\Delta f^{\mathrm{eq}}_{\alpha'}. \tag{64b}$$

Equation (64b) follows from (64a) after using Eq. (58) to express $\partial\Delta f^{\mathrm{eq}}/\partial\mathbf{k}$ in terms of f^{eq} and Δf^{eq}, and absorbing an inner sum into $(\mathrm{C}\widetilde{\mathbf{E}}{\cdot}\mathbf{v})^2$.

The term above is markedly different from the rate of energy loss $P(\mathbf{r})$ from Joule heating. By contrast, $S_g(\mathbf{r},0)$ relates directly to nonequilibrium broadening of the fluctuations, due to the kinetic energy gained during ballistic motion; the extent of the broadening is dynamically constrained by dissipation. The impact of this term on current noise is felt only for significant departures from equilibrium.

In a degenerate system there is an additional, purely kinematic, constraint on field-driven broadening, seen directly in the factor $(1-2f^{\mathrm{eq}})$ of Eq. (64b). This inhibits the contribution of S_g relative to the corresponding classical result, in which the factor is unity. Suppression of electron heating by Fermi-Dirac statistics reflects the large energy cost of displacing electrons deep inside the Fermi sea.

To highlight the difference between dissipative and hot-electron terms it is instructive to revisit a simple example,[31,41] the uniform electron gas in the constant collision time (Drude) approximation, subject to a field $\mathbf{E} = -E\hat{\mathbf{x}}$. Expressions for the power density P and hot-electron component S_g are derived in Appendix B. The thermally driven current-current spectral density, over a uniform sample of length L_x and total volume Ω, is[36]

$$\mathcal{S}(E,\omega) = 4\sum_{\mathbf{r}}\Omega(\mathbf{r})\sum_{\mathbf{r}'}\Omega(\mathbf{r}')\left\langle\!\!\left\langle \left(-\frac{ev_x}{L_x}\right)\left(-\frac{ev'_x}{L_x}\right)\Delta\mathrm{f}^{(2)}(\omega)\right\rangle\!\!\right\rangle'_c$$

$$= 4\frac{\Omega S_f(\omega)}{L_x^2 E^2}. \tag{65}$$

Introducing the conductance $\mathcal{G} = \Omega P/L_x^2 E^2$, the static limit of the spectrum is determined by Eq. (62):

$$\mathcal{S}(E,0) = 4\mathcal{G}k_BT\left[1 + \frac{S_g(0)}{Pk_BT}\right] = 4\mathcal{G}k_BT\left[1 + \frac{\Delta n}{n}\left(\frac{m^*\mu_e^2E^2}{k_BT}\right)\right]. \tag{66}$$

We have substituted for P and S_g from Eqs. (B7) and (B9). The electronic density is n while $\Delta n = \Delta N/\Omega$ is the number-fluctuation density. The effective electron mass is m^* and μ_e is the mobility.

The term S_g/Pk_BT on the right-hand side of Eq. (66) is a relative measure of the hot-electron contribution to the noise. The inhibiting effect of degeneracy, through $\Delta n/n$, is greatest at low temperature; in terms of the Fermi energy $\varepsilon_F \propto n^{2/\nu}$ we have

$$\frac{\Delta n}{n} = \frac{k_BT}{n}\frac{\partial n}{\partial \varepsilon_F} \to \frac{\nu k_BT}{2\varepsilon_F}. \tag{67}$$

When $\varepsilon_F \ll k_BT$ the ratio $\Delta n/n$ is unity; the hot-electron term is that of a classical electron gas (low density, high temperature), whose high-field behaviour is $\mathcal{S} = 4\mathcal{G}m^*\mu_e^2E^2$ independently of T. On the other hand, when $k_BT \ll \varepsilon_F$, the system is strongly degenerate and Eq. (66) with Eq. (67) yields

$$\frac{\mathcal{S}(E,0)}{4\mathcal{G}k_BT} \to 1 + \frac{\nu}{2}\left(\frac{m^*\mu_e^2E^2}{\varepsilon_F}\right). \tag{68}$$

The thermal fluctuation spectrum necessarily vanishes with temperature, but its ratio with the Johnson-Nyquist spectral density $4\mathcal{G}k_BT$ continues to exhibit a hot-electron excess, now scaled by the dominant energy ε_F. Figure 1 illustrates the behaviour of the spectral ratio for a two-dimensional electron gas, as a function of the applied field as T is taken from the degenerate limit to above the Fermi temperature $T_F = \varepsilon_F/k_B$. We see the gradual trend towards the classical form of Eq. (66) with rising temperature.

Equation (68) may be compared with a perturbative estimate by Landauer[21] for the degenerate limit, in which the analogous hot-electron contribution is $(\delta U/k_BT)^2$ where $\delta U \sim m^*\mu_e E v_F$ is a characteristic energy gain and v_F is the Fermi velocity. Taken at face value, this suggests that hot-electron effects in the low-T regime can be further enhanced by cooling. A series expansion in powers of E does not take into account non-analyticity of the Boltzmann solutions in the approach to equilibrium;[42] see also Eqs. (C9) and (C14) of our Appendix C. Non-analyticity of the distribution function $f_{\mathbf{k}}$ precludes the reliable calculation of moment averages by expanding away from equilibrium.

The relevance of non-analyticity to transport physics has been questioned by Kubo, Toda, and Hashitsume.[43] They ascribe its appearance to the simplistic treatment of real collision processes by the Drude approximation, despite strong evidence by Bakshi and Gross[42] that non-analyticity is generic to Boltzmann solutions. Even in the Drude model, the nonperturbative solution produces a physically coherent account of the temperature dependence of nonequilibrium fluctuations, while finite-order response theory does not. Such clear qualitative differences between perturbative and nonperturbative predictions should be detectable in the nonequilibrium noise.

There exist several alternative generalisations of the FD relation.[35–37] We mention the best known, which defines the nonequilibrium noise temperature T_n and is pivotal to the interpretation of device-noise data.[36] This effective Nyquist temperature is obtained, for a nonlinear operating point, by normalising $\mathcal{S}$ with the differential conductance $\mathcal{G}(E) = (e\Omega/L_x^2)\partial\langle v_x g\rangle/\partial E$ such that $T_n(E) = \mathcal{S}(E,0)/4\mathcal{G}(E)k_B$, corresponding to the output of small-signal noise measurements. Our Eqs. (60) – (64) provide a microscopic framework for computing $\mathcal{S}$ in a wide class of degenerate systems. Since $\mathcal{G}(E)$ is also calculable, this yields T_n.

B. Shot Noise

Carrier fluctuations manifest as shot noise when they are induced by random changes in the discrete flux at the terminals, rather than by thermal agitation distributed through the body of the conductor. Consider an open segment of electron gas between macroscopic leads. For this segment we add up the transient, time-of-flight correlations between the current at the source boundary, $x = x_1$, and that at the drain boundary, $x = x_2$.

The total shot noise measured across the boundaries is the resultant of two components. One component represents the response at the drain terminal to the random *entry* of electrons from the source reservoir, while the other represents the response at the source terminal to the random *exit* of electrons out to the drain reservoir. Thus

$$\mathcal{S}_{\rm sh}(|x_2 - x_1|) = \mathcal{S}_{\overleftarrow{\rm sh}}(x_2, x_1) - \mathcal{S}_{\overleftarrow{\rm sh}}(x_1, x_2) \tag{69}$$

where the unidirectional term $\mathcal{S}_{\overleftarrow{\rm sh}}(x_j, x_i)$ correlates the induced flux at x_j with the random inducing flux at x_i, and has a structure determined as follows. To begin with, note that the correlated two-particle fluctuation at time t, following a spontaneous change $\delta N_{s''}$ in the population of spin subband s'', is

$$\Big(\widetilde{R}_{\alpha\alpha'}(t) - \widetilde{R}_{\alpha\alpha'}(\infty)\Big)\delta f_{\alpha'} = \widetilde{C}_{\alpha\alpha'}(t)\frac{\delta f_{\alpha'}}{\delta N_{s''}}\delta N_{s''}.$$

Coulomb screening is fully incorporated. When a particle is added at x_1 we have $\delta N_{s''} = +1$; when a particle is removed at x_2, then $\delta N_{s''} = -1$. The sign of $\delta N_{s''}$ determines the sign of the corresponding unidirectional term in Eq. (69).

We next observe that, for each of the active carriers in the segment's population $N = \sum_{s''} N_{s''}$, the time of arrival at the source boundary is uncorrelated with all other arrival times. (In the same way, departure times at the drain are mutually uncorrelated.) It follows that $\mathcal{S}_{\overleftarrow{\rm sh}}$ is an incoherent sum of transients:

$$\begin{aligned}
\mathcal{S}_{\overleftarrow{\rm sh}}(x_j, x_i) &\stackrel{\rm def}{=} 2e^2 \sum_{s''} N_{s''} \Bigg\{ \sum_{\alpha[x=x_j]} \sum_{\alpha'[x'=x_i]} \int_0^\infty dt\ (v_x)_{\mathbf{k}} \widetilde{C}_{\alpha\alpha'}(t) (v_x)_{\mathbf{k}'} \frac{\delta f_{\alpha'}}{\delta N_{s''}} |\delta N_{s''}| \Bigg\} \\
&= 2e^2 \frac{N}{\Delta N} \int d^\nu r \delta(x - x_j) \int d^\nu r' \delta(x' - x_i) \frac{1}{\gamma_C} \langle\!\langle v_x v'_x \widetilde{\Delta}\mathrm{f}^{(2)}(\mathbf{r}, \mathbf{r}'; 0) \rangle\!\rangle'_c,
\end{aligned} \tag{70}$$

where we have used Eq. (49) with the spin trace

$$\sum_{s''} N_{s''} \frac{\delta f_{\alpha'}}{\delta N_{s''}} = \frac{N}{2} \sum_{s''} \frac{\delta f_{\alpha'}}{\delta N} \frac{\delta N}{\delta N_{s''}} = \frac{N \widetilde{\Delta} f_{\alpha'}}{\widetilde{\Delta} N}.$$

Our many-body construction of shot noise rests on two assumptions. The first is ergodicity, after the original argument of Schottky: a typical carrier in the segment must enter through the source (cathode) and, eventually, leave through the drain (anode). There is no temporal correlation among individual transits. The second assumption is that each carrier in the ensemble has distinct roles as both agent and spectator: it generates shot noise, and it is also part of the many-body response making up the shot noise. These dual roles are statistically independent.

The phenomenological content of Eqs. (69) and (70) is the same as for the Boltzmann-Langevin formalism[19] except that there is no longer any need for commitment to a specific collisional form (other than expecting it to satisfy conservation). Notice too that the total shot noise vanishes identically at equilibrium because detailed balance renders the equation of motion for the resolvent $R^{\rm eq}(t)$ collisionless, and hence self-adjoint. Self-adjointness is preserved for $\widetilde{R}^{\rm eq}(t)$ since Coulomb forces are conservative. One can then show that

$$\langle\!\langle v_x \mathrm{Re}\{\widetilde{\mathrm{C}}^{\rm eq}_{\alpha\alpha'}(\omega)\} v'_x \widetilde{\Delta} f^{\rm eq}(\mathbf{r}') \rangle\!\rangle' \equiv \langle\!\langle \widetilde{\Delta} f^{\rm eq}(\mathbf{r}) v_x \mathrm{Re}\{\widetilde{\mathrm{C}}^{\rm eq}_{\alpha\alpha'}(-\omega)\} v'_x \rangle\!\rangle'.$$

This produces exact cancellation between the right-hand terms of Eq. (69).

Significantly, the Thomas-Fermi screening coefficient γ_C dividing the flux autocorrelation in the second line of Eq. (70), is cancelled exactly by its presence within the autocorrelation via Eqs. (52) and (56). This means that the only type of Coulomb screening affecting the shot noise is collision-mediated. Indeed, any homogeneous renormalisation of the equilibrium fluctuations leaves the ratio $\Delta f/\Delta N$ untouched. Consequently such a rescaling has absolutely no influence on the shot noise. By comparison, the effect on the free-electron Johnson noise can be dramatic[31] since it is proportional to γ_C.

The contrast between their Coulomb responses is one demonstration that thermal noise and shot noise are in fact distinct many-body phenomena. Although they share a common microscopic structure in Eq. (56), in thermodynamic terms one is an extensive continuum quantity driven by fluctuations of the kinetic energy, while the other is short-ranged and corpuscular, driven by fluctuations of the local particle number. For strongly degenerate systems the two are disproportionate because of the scale difference $N/\widetilde{\Delta}N$, which becomes unity only in the classical limit. It follows that, unlike the perturbative treatments,[11] this formalism will not admit a universal interpolation formula giving thermal noise in one regime and (true) shot noise elsewhere.[44]

We propose a simple experimental test of incommensurability, applicable at any current. In a point-contact constriction defined on a two-dimensional electron gas at a heterojunction, thermal and shot noise are both measurable.[4,5] Thermal noise, by its scaling with γ_C, depends strongly on electron density.[31] Shot noise does not share this dependence. If the density in the channel is changed, for example by back-gate biasing, the thermal noise should vary

strongly with the bias voltage. By contrast, the shot noise should have none of this variation since it is immune to self-screening of the carrier fluctuations in the quantum well.

Our direct concern is the high-field limit, where inelastic collisions rule. For an initial look at high-field shot noise we explore the Drude model of a uniform wire, emphasising that its low-field behaviour, though revealing, does not address the elastically dominated diffusive regime.[11] We take the case where the segment and its leads are physically identical.[30] This is an idealised example; experimentally there must be some differentiation between sample and reservoirs, expressible in known boundary conditions.

The region has length $l = x_2 - x_1 \ll L_x$, where L_x is the length of the complete assembly, segment plus leads. Since at least one of the dimensions may approach the mean free path, the BGFs for this problem have the short-range spatial structure detailed in Appendix C and modified by collisional Coulomb effects. While we do not analyse the latter in computational detail here, we propose a useful approximation based on the Ansatz of Eq. (A4) for the operator Γ applied to Eqs. (52) and (55). In a conductor of diameter much greater than the mean free path, we take the bulk Fourier coefficients $\mathrm{C}^{(b)}(\mathbf{q},0)$ and $\gamma_{\rm coll}(\mathbf{q},0)$, respectively for the correlated propagator of Eq. (C12) and the collision-mediated Coulomb suppression of Eq. (A2). The unidirectional shot noise of Eq. (70) then becomes

$$\begin{aligned}\mathcal{S}_{\overleftarrow{\rm sh}}(x_j,x_i) \approx 2\frac{ne^2}{\Delta n}\iint d^{\nu-1}r_\perp \iint d^{\nu-1}r'_\perp \int \frac{d^\nu q}{(2\pi)^\nu}\exp\{i[q_x(x_j-x_i)+\mathbf{q}_\perp\cdot(\mathbf{r}_\perp-\mathbf{r'}_\perp)]\}\\ \times\gamma^2_{\rm coll}(\mathbf{q},0)\int\frac{2d^\nu k}{(2\pi)^\nu}\int\frac{d^\nu k'}{(2\pi)^\nu}(v_x)_{\mathbf{k}}\mathrm{C}^{(b)}_{\mathbf{kk'}}(\mathbf{q},0)(v_x)_{\mathbf{k'}}\Delta f_{\mathbf{k'}}\end{aligned} \tag{71}$$

where, for any wave vector $\mathbf{u}$, we write its transverse component as $\mathbf{u}_\perp$. We have dropped a contribution $\sim \gamma^3_{\rm coll}$ coming from the second term on the right-hand side of Eq. (55) for $\widetilde{\mathrm{C}}$. In a full study of nonequilibrium Coulomb processes within shot noise, $\gamma_{\rm coll}$ is clearly central. Note that there are no transverse terms in one dimension (1D), nor is it strictly possible to discuss semiclassical Coulomb effects in 1D.

IV. CALCULATIONS

In this Section we present calculations for our inelastic model. We omit Coulomb screening, letting $\gamma_{\rm coll}\to 1$ in Eq. (71). To build up a detailed picture we start with shot noise in a one-dimensional wire.

A. One Dimension

For a segment much shorter than the total system size, we can simplify the calculation by setting $\mathrm{C}^{(b)}(q,0)\approx \mathrm{C}^{(0)}(q,0)$; see Eqs. (C11) and (C12). The omitted term is proportional to the finite resolution function $\varphi_1(q;\frac{1}{2}L_x)\sim \delta(q)/L_x$. Since, over the segment, most of the structure involves $ql \gtrsim 1$, the approximation results in a negligible error of order $l/L_x \ll 1$.

Proceeding from Eq. (71) with $\nu = 1$ and using Eq. (C9b) for $\mathrm{C}^{(0)}$ we obtain

$$\begin{aligned}\mathcal{S}_{\overleftarrow{\rm sh}}(\xi) &= 2\frac{ne^2}{l\Delta n}\left(\frac{\hbar}{m^*}\right)^2\int k\frac{dk}{\pi}\int k'dk'\frac{\tau}{k_d}\theta(k-k')e^{-(k-k')/k_d}\Delta f_{k'}\\ &\quad\times\int l\frac{dq}{2\pi}\exp\Big[iq\Big(\xi l-\frac{\hbar\tau}{2m^*k_d}(k^2-k'^2)\Big)\Big]\\ &= 2\frac{ne^2\tau}{m^*l}\left(\frac{\hbar^2}{m^*\pi\Delta n}\right)\frac{1}{k_d}\int kdke^{-k/k_d}\int_{-\infty}^{k}k'dk'e^{k'/k_d}\Delta f_{k'}\delta\Big(\xi-(k^2-k'^2)/p_d^2\Big).\end{aligned} \tag{72}$$

We have introduced $\xi = (x_j - x_i)/l$ for $i, j = 1, 2$ and the wave number p_d defined by $p_d^2 = 2m^*lk_d/\hbar\tau = 2m^*eV/\hbar^2$. Next, use the expression for Δf in terms of $\Delta f^{\rm eq}$; see Eq. (C14a). Rearranging the order of integration, we get

$$\begin{aligned}\mathcal{S}_{\overleftarrow{\rm sh}}(\xi) &= 2\frac{ne^2\tau}{m^*l}\left(\frac{\hbar^2p_d^2}{2m^*}\right)\int\frac{dk''}{\pi}\frac{\Delta f^{\rm eq}_{k''}}{\Delta n}e^{k''/k_d}\int_{k''}^{\infty}\frac{kdk}{k_d^2}e^{-k/k_d}\int_{|k''|}^{|k|}2k'dk'\delta(k'^2+\xi p_d^2-k^2)\\ &= 2eI\int_0^\infty\frac{dk''}{\pi}\frac{\Delta f^{\rm eq}_{k''}}{\Delta n}\Bigg\{e^{k''/k_d}\int_{k''}^{\infty}\frac{kdk}{k_d^2}e^{-k/k_d}\theta(\xi)\theta\Big(k-\sqrt{k''^2+p_d^2}\Big)\end{aligned}$$

$$+ e^{-k''/k_d} \int_{-k''}^{\infty} \frac{kdk}{k_d^2} e^{-k/k_d} \left[\theta(\xi)\theta\Big(k - \sqrt{k''^2 + p_d^2}\Big) - \theta(-\xi)\theta\Big(k'' - \sqrt{k^2 + p_d^2}\Big) \right] \Bigg\} \tag{73}$$

where $I = (ne^2\tau/m^*l)V$ is the current through the wire. We now go to the degenerate limit, replacing the equilibrium fluctuation according to $\Delta f_k^{\rm eq} = (m^* k_B T/\hbar^2 k_F)\delta(|k| - k_F)$. The number-fluctuation density is $\Delta n = 2m^* k_B T/\hbar^2 \pi k_F$. A series of manipulations leads to the total shot noise

$$\begin{aligned}
\mathcal{S}_{\rm sh}(l) &= \mathcal{S}_{\overleftarrow{\rm sh}}(+1) - \mathcal{S}_{\overleftarrow{\rm sh}}(-1) \\
&= 2eI \Bigg\{ e^{-\sqrt{k_F^2 + p_d^2}/k_d} \left[1 + \frac{\sqrt{k_F^2 + p_d^2}}{k_d} \right] \cosh\left(\frac{k_F}{k_d}\right) - \theta(\varepsilon_F - eV) e^{-k_F/k_d} \\
&\quad \times \left[\frac{\sqrt{k_F^2 - p_d^2}}{k_d} \cosh\left(\frac{\sqrt{k_F^2 - p_d^2}}{k_d} \right) - \sinh\left(\frac{\sqrt{k_F^2 - p_d^2}}{k_d} \right) \right] \Bigg\}.
\end{aligned} \tag{74}$$

Again we see the non-analyticity of this expression with respect to the driving field. In this uniformly-embedded wire model, expansion of the shot noise in powers of V is not valid at low fields. On the contrary, the shot noise becomes perturbative in $1/V$ at high fields, which could be termed the Schottky domain. The expression is perfectly calculable and there are two asymptotic cases of interest.

(a) High fields, $eV \gg \varepsilon_F$:

$$\begin{aligned}
\mathcal{S}_{\rm sh}(l)\Big|_{V\to\infty} &= 2eI\left(1 + \frac{p_d}{k_d}\right) e^{-p_d/k_d} \\
&\to 2eI\left(1 - \frac{m^*(l/\tau)^2}{eV}\right).
\end{aligned} \tag{75}$$

The high-field limit gives the full Schottky expression $2eI$ with a correction, dependent on the wire parameters, which is asymptotically negligible. The same formal result also holds for any chosen value of I in the collisionless regime $\tau \to \infty$, confirming that our model recovers the shot-noise behaviour of a monoenergetic flux.

(b) Low fields, $eV \ll \varepsilon_F$. The mean free path is $\lambda = \tau v_F$. Then $(k_F^2 \pm p_d^2)^{\frac{1}{2}} \to k_F[1 \pm lk_d/\lambda k_F - \frac{1}{2}(lk_d/\lambda k_F)^2]$ and

$$\mathcal{S}_{\rm sh}(l)\Big|_{V\to 0} = 2eI\left(1 + \frac{l}{\lambda} + \frac{l^2}{2\lambda^2}\right) e^{-l/\lambda}. \tag{76}$$

Our 1D inelastic model again gives $2eI$ at low fields, with no suppression in the ballistic limit $\lambda \gg l$. There is, hovever, exponential decay of the shot noise as the wire length increases beyond the mean free path. The result is understandable as source and drain currents rapidly decorrelate with increasing l/λ; if $\lambda \propto \tau$ is made smaller while l is kept fixed, the exponential attenuation is broadly consistent with Monte Carlo results of Liu, Eastman, and Yamamoto.[22] Their more general simulation of 1D low-field shot noise, which includes both elastic scattering and inelastic phonon emission, exhibits strong suppression in the inelastically dominated regime.

In Fig. 2 we compare the 1D shot noise normalised to $2eI$, for degenerate and classical conductors. Fig. 2(a) shows the results for a degenerate sample, as a function of current normalised to $I_F = nev_F$. The plots are for a range of wire lengths from the ballistic limit $l = 10^{-6}\lambda$, up to $l = 10\lambda$. At low fields the intercepts at $I = 0$, given by Eq. (76), show their attenuation away from the ballistic limit. For higher currents, the shot noise quickly settles to the form given by Eq. (75).

In Fig. 2(b) we plot, for comparison, the shot noise of a 1D system whose carrier distribution is classical: $\Delta f_k^{\rm eq} = f_k^{\rm eq} \propto \exp(-\varepsilon_k/m^* v_{\rm th}^2)$ where $v_{\rm th} = (k_B T/m^*)^{\frac{1}{2}}$. The current is normalised to $I_{\rm th} = ne\sqrt{2}v_{\rm th}$ and $\lambda = v_{\rm th}\tau$.. Also plotted is the asymptotic form appearing in the first line of Eq. (75). Again at low currents we see attenuation with increasing wire length, stronger than in Fig. 2(a). At higher currents there is the same rapid convergence to the asymptotic result (evident at surprisingly modest currents) as found in Fig. 2(a).

From our comparison of Fermi-Dirac and Maxwell-Boltzmann versions of the model, we conclude that degeneracy contributes mainly at currents below I_F. For $eV < \varepsilon_F$ the driving voltage cannot overcome the collective stability of the Fermi sea, and the zero-current correlations persist. For $eV \gtrsim \varepsilon_F$ appreciable redistribution of the particle occupancies suddenly becomes possible, with an initial dip in relative correlation strength. At higher fields most electrons move independently and ballistically, in the sense that $\tau\langle vf\rangle/n \gg l$. The shot noise is then in the Schottky domain.

B. Two and Three Dimensions

For higher dimensions we must include traces over the transverse degrees of freedom. Write $A \equiv 2R$ for $\nu = 2$ and $A \equiv \pi R^2$ for $\nu = 3$ where R is the half-width (for a strip) or the radius (for a cylinder). As in the 1D case, the condition $l \ll L_x$ implies that the correlated BGF is well approximated by

$$\mathrm{C}_{\mathbf{k}\mathbf{k}'}(\mathbf{q}, \mathbf{q}', \omega) = A\delta(q_x - q'_x)\varphi_{\nu-1}(\mathbf{q}_\perp - \mathbf{q}'_\perp; R)\mathrm{C}^{(0)}_{\mathbf{k}\mathbf{k}'}(\mathbf{q}, \omega);$$

refer to Appendix C for details. After integrating over the cross-sectional co-ordinates and applying Eq. (C9b) for $\mathrm{C}^{(0)}$, Eq. (70) reads

$$\begin{aligned} \mathcal{S}_{\overleftarrow{\mathrm{sh}}}(\xi) = 2\frac{ne^2A}{l\Delta n}\Big(\frac{\hbar}{m^*}\Big)^2 \int \frac{2k_x dk_x}{(2\pi)^\nu} \int k'_x dk'_x \frac{\tau}{k_d}\theta(k_x - k'_x)e^{-(k_x - k'_x)/k_d} \\ \times \int d^{\nu-1}k_\perp \Delta f_{\mathbf{k}'} F_\nu(a) \int l\frac{dq}{2\pi} \exp\Big[ilq\Big(\xi - p_d^{-2}(k_x^2 - k'^2_x)\Big)\Big]. \end{aligned} \tag{77}$$

The shape factor $F_\nu(a)$, whose argument is $a = \hbar\tau|k_x - k'_x|k_\perp/(2m^*k_dR)$, has the form

$$\begin{aligned} F_\nu(a) = A^2 \int \frac{d^{\nu-1}q_\perp}{(2\pi)^{\nu-1}}\varphi_{\nu-1}(\mathbf{q}_\perp; R)\exp\Big(-\frac{i\hbar\tau(k_x - k'_x)}{m^*k_d}\mathbf{k}_\perp\cdot\mathbf{q}_\perp\Big) \\ \times \int \frac{d^{\nu-1}q'_\perp}{(2\pi)^{\nu-1}}\varphi_{\nu-1}(\mathbf{q}_\perp - \mathbf{q}'_\perp; R)\varphi_{\nu-1}(\mathbf{q}'_\perp; R), \end{aligned} \tag{78a}$$

which reduces to the expressions

$$F_2(a) = \theta(1 - a)\Big(1 - a\Big), \tag{78b}$$

$$F_3(a) = \theta(1 - a)\Big[1 - \frac{2}{\pi}\Big(\arcsin a + a\sqrt{1 - a^2}\Big)\Big]. \tag{78c}$$

In a wire of finite width, this function directly expresses the constraint on lateral motion of the carriers; it cross-couples, kinematically, the transverse and longitudinal modes. Here there is none of the dynamical cross-coupling induced, for example, by elastic scattering. Kinematic suppression is inherent in the form of $\mathrm{C}^{(0)}$ [Eqs. (C8) and (C9)], itself conditioned by the free-streaming operator in the Boltzmann equation. It is not surprising to find an entirely geometric source of shot-noise suppression in two and three dimensions (2D; 3D). This echoes, in part, Landauer's remark[23] on the need to sample more than just the longitudinal trajectories in any semiclassical calculation.

We process Eq. (77) along lines analogous to Eq. (73), going to the degenerate limit with Eq. (C14b) for Δf and the fluctuation density $\Delta n = m^*k_BTk_F^{\nu-2}/\hbar^2\pi^{\nu-1}$. The first shot-noise component reduces to

$$\begin{aligned} \mathcal{S}_{\overleftarrow{\mathrm{sh}}}(+1) = 2eI\int_{p_d}^{\infty} \frac{k_x dk_x}{k_d^2}e^{-k_x/k_d}\int_0^{k_F}\Big(\frac{2k_F}{\pi k_\perp}\Big)^{3-\nu}\frac{dp_\perp}{2k_F} \\ \times\Bigg[2\theta(p_x - p_\perp)\cosh\Big(\frac{p_\perp}{k_d}\Big)F_\nu(a_-) \\ +\theta(p_\perp - p_x)e^{-p_\perp/k_d}\Big(F_\nu(a_-) - F_\nu(a_+)\Big)\Bigg] \end{aligned} \tag{79a}$$

where $p_\perp = (k_F^2 - k_\perp^2)^{\frac{1}{2}}$ and $p_x = (k_x^2 - p_d^2)^{\frac{1}{2}}$; the shape-factor arguments are $a_\pm = \hbar\tau(k_x \pm p_x)k_\perp/(2m^*k_dR)$. Similarly the second component is

$$\begin{aligned} \mathcal{S}_{\overleftarrow{\mathrm{sh}}}(-1) = 2eI\theta(\varepsilon_F - eV)\int_{p_d}^{k_F}\frac{k_x dk_x}{k_d^2}\int_{k_x}^{k_F}\Big(\frac{2k_F}{\pi k_\perp}\Big)^{3-\nu}\frac{dp_\perp}{2k_F}e^{-p_\perp/k_d} \\ \times\Big[e^{p_x/k_d}F_\nu(a_-) - e^{-p_x/k_d}F_\nu(a_+)\Big]. \end{aligned} \tag{79b}$$

In Figs. 3 and 4 we plot the shot noise in two and three dimensions for a range of wire geometries, as a function of current normalised to $I_F = neAv_F$. The results in Fig. 3 are calculated for the thick-wire limit $R \gg \lambda$; those in Fig. 4 are for a thin wire, $R = 0.3\lambda$. The same values of λ/l are used as in Fig. 2, running down monotonically from the top.

For both strips and cylinders in Fig. 3, it is the behaviour of $\mathcal{S}_{\rm sh}$ at higher values of I that comes to notice first: each curve merges with its asymptotic 1D analogue illustrated in Fig. 2(b). As will shortly become clear, it is only in the thick-wire limit, and *then* only for high currents, that a 1D treatment can in any sense mimic the exact calculations for higher dimensions [those conditions amount to setting $F_\nu \approx 1$ within Eq. (79)]. In fact, as one progresses from 1D through 3D [Figs. 2(a), 3(a), and 3(b)], the zero-current intercepts of the curves indexed by the same λ/l undergo a marked and systematic increase in suppression. This shows (at least in the simple Drude model) that the 1D calculation is a poor estimate of low-field noise in realistic geometries, even in the thick-wire limit.

We come to the thin wires of Fig. 4. The uppermost curves, in the ballistic regime $\lambda \gg l$, are unchanged from Figs. 2 and 3. However, relative to Fig. 3 the remaining curves in Fig. 4 change dramatically as one moves further from the ballistic limit. There is now substantial shot-noise suppression throughout the whole range of I. For example, in Fig. 3(b) the longest 3D wire, $l = 10\lambda$, has $\mathcal{S}_{\rm sh}/2eI = 0.59$ at the highest current $I = 10I_F$, while its opposite number in Fig. 4(b) reaches the value 0.2; a threefold reduction. Calculations at much higher fields confirm the eventual recovery of full shot noise in keeping with Eq. (75).

The outcome of spatially constrained carrier motion is the extensive suppression of shot noise over a wide range of the current. We stress that the effect is implicit in the generic structure of the Boltzmann equation, and its propagators, for 2D and 3D [see Eq. (C8)]; it is simply absent in 1D. Therefore, this mode of suppression cannot be simulated by any one-dimensional scheme.

In a 3D wire the Fermi wavelength is 0.05 nm at metallic electron densities. If $\lambda = 50$ nm, typical for strong inelastic scattering, a wire of width ~ 30 nm would exhibit kinematic shot-noise suppression at large currents, providing that it was not masked by collisional Coulomb effects. In future we plan to assess the latter quantitatively; the comparative action of nonequilibrium screening, in 2D versus 3D, should itself be an interesting window on how dimensionality affects fluctuations.[27]

V. SUMMARY

We have described and implemented a nonperturbative microscopic formalism for current fluctuations in metallic systems, down to the mesoscopic scale, within the ambit of semiclassical theory. Our strategy for incorporating fermion correlations into the Boltzmann picture safeguards the conservation laws at both the single-particle level and at the level of dynamic two-particle processes, the key to nonequilibrium current noise. In particular we have derived the nonlinear analogue of the fluctuation-dissipation theorem. It should also be straightforward to include semiclassical electron-electron scattering in our description of transport and noise.

Our formalism's calculability stems from the flexible structure of the Boltzmann-Green functions. These serve as semiclassical propagators of the electronic Fermi-liquid correlations, mapping them uniquely to the correlations of the nonequilibrium system. We have demonstrated their usefulness in shedding light on the physics of high-current shot noise, and on the importance of treating dimensionality correctly in constricted mesoscopic samples.

The present account of nonequilibrium fluctuations raises a variety of interesting questions, practical and abstract. By far the most salient is the relation between thermal noise and shot noise; our claim that the two are thermodynamically incommensurate should be easy to test. Thermal-noise measurements on a gated two-dimensional mesoscopic wire, defined on a III-V heterojunction, ought to give a strong gate-voltage-dependent signature of Thomas-Fermi suppression from self-confinement of the carriers in their quantum well. Measurements of the shot noise in the same structure should give no such signature.

We end with just two out of many theoretical issues. The first is the role of non-analyticity of the BGF solutions at low fields, and the implications for semiclassical linear response. The phenomenon is well known in uniform systems,[30,42] where the free-streaming part of the Boltzmann operator is manifestly anomalous in its vanishing with the applied field. While we have no firm information on whether the BTE for nonuniform systems shares this behaviour, we point out that our shot-noise results, obtained in a spatially inhomogeneous model (albeit weakly so), are certainly non-analytic in the applied voltage. The low-field asymptotics of the general Boltzmann equation have an obvious bearing on how semiclassical noise is to be calculated, and their clarification would be a significant advance.

Second, there is the status of the BGF approach within quantum kinetics. In terms of, say, the Kadanoff-Baym analysis of the Boltzmann equation,[45] our semiclassical equation of motion for the fluctuations should emerge from the quantum evolution of the particle-hole amplitudes in the long-wavelength limit,[46] much as the ordinary BTE is distilled from the long-wavelength dynamics of the density matrix for a single particle.

ACKNOWLEDGMENTS

We are indebted to the late R. Landauer for his encouragement, which stimulated the present work in its earliest stages. We thank E. Davies for help with calculations and figures and N. W. Ashcroft, R. J-M. Grognard, and D. Neilson for fruitful discussions.

Last, but not least, we acknowledge the cultural contribution of Physical Review B and its editors. But for that journal's egregious year-long dissembling over its submission, this work might still be written in American English.

APPENDIX A: TYPES OF SCREENING IN NONEQUILIBRIUM SYSTEMS

We begin this Appendix with some basic properties of collision-mediated Coulomb screening, which influences both thermal and shot noise. We end with Thomas-Fermi screening, which influences thermal noise alone. Keeping in mind the high-field application, we base our calculations on the collision time model (see the following Appendices). Our results are specific to a bulk system with inelastic scattering; we stress both their illustrative intent and their inapplicability to the elastic diffusive regime, where the Boltzmann propagators and the collisional screening effects are very different.

We take the Fourier transform of the collision-mediated Coulomb operator [see Eq. (43)] using the form of the correlated BGF in the bulk limit, Eq. (C12). The (spin-independent) function Γ is

$$\Gamma_{\mathbf{kk'}}(\mathbf{q};\omega) = \Omega\delta_{\mathbf{kk'}} - \frac{2e}{\hbar\Omega^2}\sum_{\mathbf{k''}} \mathrm{C}^{(b)}_{\mathbf{kk''}}(\mathbf{q};\omega)\frac{\partial f_{\mathbf{k''}}}{\partial \mathbf{k''}}\cdot\boldsymbol{\mathcal{E}}_C(\mathbf{q})\sum_{\mathbf{k'''}}\Gamma_{\mathbf{k'''k'}}(\mathbf{q};\omega), \tag{A1}$$

where the electron field is $e\boldsymbol{\mathcal{E}}_C(\mathbf{q}) = -i\mathbf{q}\mathrm{V}_C(q)$ and $\mathrm{V}_C(q)$ is the Coulomb potential transform. In three dimensions there is a complication owing to the long-range Coulomb tail; by assumption, all fields are shorted out beyond the system boundaries. We model this constraint by introducing a cutoff, so that $\mathrm{V}_C(q) = 4\pi e^2/\epsilon(q^2+\kappa^2)$ where $\kappa^{-1} \gtrsim \Omega^{\frac{1}{3}}$ represents the characteristic length scale beyond which the fields are zero.

The trace $\gamma_{\rm coll}(\mathbf{q},\omega) \equiv \langle\Gamma(\mathbf{q},\omega;\mathbf{k'})\rangle$ is independent of wave vector $\mathbf{k'}$. This follows from the decoupling of the internal summations over kinematic variables in Eq. (A1), since the Coulomb field depends only on $\mathbf{q}$. Summing both sides of Eq. (A1) we get

$$\begin{aligned}\gamma_{\rm coll}(\mathbf{q},\omega) &= 1 - \frac{2e}{\hbar\Omega^2}\sum_{\mathbf{k}}\sum_{\mathbf{k'}} \mathrm{C}^{(b)}_{\mathbf{kk'}}(\mathbf{q},\omega)\frac{\partial f_{\mathbf{k'}}}{\partial \mathbf{k'}}\cdot\boldsymbol{\mathcal{E}}_C(\mathbf{q})\gamma_{\rm coll}(\mathbf{q},\omega)\\ &= \left[1 - \frac{2i\mathrm{V}_C(q)}{\hbar\Omega}\sum_{\mathbf{k'}}\langle \mathrm{C}^{(b)}(\mathbf{q},\omega;\mathbf{k'})\rangle\mathbf{q}\cdot\frac{\partial f_{\mathbf{k'}}}{\partial \mathbf{k'}}\right]^{-1}.\end{aligned} \tag{A2}$$

There is now a closed form for the Coulomb operator:

$$\Gamma_{\mathbf{kk'}}(\mathbf{q},\omega) = \Omega\delta_{\mathbf{kk'}} + i\frac{8\pi e^2\gamma_{\rm coll}(\mathbf{q},\omega)}{\epsilon\hbar\Omega(q^2+\kappa^2)}\sum_{\mathbf{k''}} \mathrm{C}^{(b)}_{\mathbf{kk''}}(\mathbf{q},\omega)\mathbf{q}\cdot\frac{\partial f_{\mathbf{k''}}}{\partial \mathbf{k''}}, \tag{A3}$$

suggesting a possible approximation for the convolution of Γ with a typical distribution F, namely

$$\frac{1}{\Omega}\sum_{\mathbf{k'}}\Gamma_{\mathbf{kk'}}(\mathbf{q},\omega)F_{\mathbf{k'}} \approx \gamma_{\rm coll}(\mathbf{q},\omega)F_{\mathbf{k}}. \tag{A4}$$

This is exact in the $q \to 0$ limit and also reproduces the exact relation $\langle\langle\Gamma(\mathbf{q},\omega)F'\rangle\rangle' = \gamma_{\rm coll}(\mathbf{q},\omega)\langle F\rangle$. The Ansatz, which amounts to the decoupling $\mathrm{C}^{(b)}_{\mathbf{kk'}}/\langle \mathrm{C}^{(b)}(\mathbf{k'})\rangle \sim F_{\mathbf{k}}/\langle F\rangle$, tends to wash out the sharp features of the integrand in Eq. (A3).

We evaluate Eq. (A2) for the Drude model in the zero-field limit. Using Eq. (C13), the trace of the correlated BGF over its leading wave vector is

$$\langle \mathrm{C}^{(b)}(\mathbf{q},0;\mathbf{k'})\rangle\Big|_{E\to 0} = \frac{1}{\dfrac{i\hbar}{m^*}\mathbf{q}\cdot\mathbf{k'} + \tau^{-1}} - \frac{\varphi_3(\mathbf{q})}{\frac{1}{2}n\Omega}\sum_{\mathbf{k}}\frac{f^{\rm eq}_{\mathbf{k}}}{\dfrac{i\hbar}{m^*}\mathbf{q}\cdot\mathbf{k} + \tau^{-1}}. \tag{A5}$$

There is no contribution to Eq. (A2) from the second right-hand term of Eq. (A5), and so

$$\gamma_{\rm coll}(q,0)\Big|_{E\to 0} = \left[1 + \frac{q_{\rm TF}^2}{q^2+\kappa^2}\left(1 - \frac{\arctan(\lambda q)}{\lambda q}\right)\right]^{-1}, \tag{A6}$$

where the Thomas-Fermi wave vector is defined by $q_{\rm TF}^2 = 4\pi e^2 \Delta n/\epsilon k_B T = 4e^2 m^* k_F/\pi\epsilon\hbar^2$, and $\lambda = \tau v_F$ is the mean free path. In the small-q limit the suppression factor becomes

$$\gamma_{\rm coll}(q,0)\Big|_{E\to 0} = \frac{\kappa^2+q^2}{\kappa^2 + (1+4\pi\sigma\tau/\epsilon)q^2} \to 1 \tag{A7}$$

(σ is the conductivity), showing that there is no long-range suppression from collision-mediated screening if the asymptotic state of the leads pins the electric fields to zero there. Put differently, microscopic scattering preserves global neutrality; Thomas-Fermi screening may not, since it redistributes charge. This calls for the inclusion of buffer zones at the system boundaries (in practice, several units of $q_{\rm TF}^{-1}$) to ensure that the fields beyond the system remain evanescent.

Mesoscopically, collisional screening should be significant. In a metal $\gamma_{\rm coll}(q,0)$ can be very small; for example, in silver at 77 K its minimum is roughly 10^{-7} and its value does not rise to one half until $q^{-1} = q_{\rm TF}^{-1} = 0.06$ nm, that is, far below any mean free path and out of the semiclassical domain. In heavily doped GaAs with carrier density $n = 10^{18}$ cm^{-3}, comparable figures are $\gamma_{\rm coll} = 0.01$ for the minimum and $\gamma_{\rm coll} = 0.5$ at $q^{-1} = 10$ nm. While these are guideline figures for a simple model in its approach to equilibrium, they hint at a strong role for collisional screening suppression in high-field shot noise.

We return to Thomas-Fermi screening, associated with the contact potentials. This is a primary source of thermal-noise suppression, a thermodynamic effect free of any collision processes. We outline its behaviour in a bulk jellium conductor contacted by leads made of different jellium. The combined system is treated as an electron gas closed with respect to carrier exchange, and satisfying periodic boundary conditions.[2] We must also take explicit account of the reservoir's electrostatic potential u_r.

If the subsystems are macroscopic, a term such as the second one in the right-hand-side sum of Eq. (41) goes to its asymptotic mean u_c; the third term is negligible to the same order. Including the explicit offset from u_r, Eq. (39) generalises to

$$\begin{aligned}
\gamma_C(q) &= 1 - \frac{\delta}{\delta\mu}(u_c - u_r) \\
&= 1 - \frac{({\rm V}_C(q)n + u_c)\Omega\widetilde{\Delta}n(q)}{Nk_BT} + \frac{({\rm V}'_C(q)n' + u_r)\Omega\widetilde{\Delta}n'(q)}{N'k_BT} \\
&= 1 - \left(1 + \frac{u_c}{n{\rm V}_C(q)}\right){\rm V}_C(q)\frac{\gamma_C(q)\Delta n(q)}{k_BT} + \left(1 + \frac{u_r}{n'{\rm V}'_C(q)}\right){\rm V}'_C(q)\frac{\gamma'_C(q)\Delta n'(q)}{k_BT},
\end{aligned} \tag{A8}$$

where all primed quantities refer to the reservoir. A complementary relation holds for $\gamma'_C(q)$. Consider the conductor. In the long-wavelength limit we have

$${\rm V}_C(q)\frac{\Delta n(q)}{k_BT} = \frac{q_{\rm TF}^2}{q^2}.$$

Putting this into Eq. (A8) together with its reservoir counterpart, and taking $q \ll q_{\rm TF}, q'_{\rm TF}$, we obtain the coupled equations

$$\left(q_{\rm TF}^2 + \frac{u_c\Delta n}{k_BTn}q^2\right)\gamma_C(q) - \left(q'^2_{\rm TF} + \frac{u_r\Delta n'}{k_BTn'}q^2\right)\gamma'_C(q) = q^2; \qquad \gamma_C(q) + \gamma'_C(q) = 2, \tag{A9}$$

giving the limiting solutions

$$\gamma_C = 1 - \frac{q_{\rm TF}^2 - {q'}_{\rm TF}^2}{q_{\rm TF}^2 + {q'}_{\rm TF}^2} \quad \text{and} \quad \gamma'_C = 1 + \frac{q_{\rm TF}^2 - {q'}_{\rm TF}^2}{q_{\rm TF}^2 + {q'}_{\rm TF}^2}. \tag{A10}$$

In this system, a conductor that is more metallic than the reservoir has $q_{\rm TF} > q'_{\rm TF}$, leading to $\gamma_C < 1 < \gamma'_C < 2$. Its thermal noise (were it accessible from outside) would thus undergo suppression. Conversely, a relatively less metallic sample would display *enhanced* thermal noise. If $q_{\rm TF} \ll q'_{\rm TF}$, as in a lightly doped bulk semiconductor in contact with a metal, then γ_C approaches its maximum of two.

A closed model provides an unrealistic picture of actual transport; it may be indicative, but by no means definitive. In particular, for truly open reservoirs, the potential u_r cannot be accessed in isolation from the chemical potential as a whole. This means that the reciprocity between γ_C and γ'_C, characteristic of the closed model, does not hold. Instead, the identity $\gamma'_C \equiv 1$ is a necessary boundary constraint.

For an open mesoscopic conductor, details of the contact-potential effects on thermal noise will be sensitive both to the topology of its boundary conditions, and to its internal electronic structure. The action of suppression (or indeed of enhancement) is a problem for further study.

APPENDIX B: UNIFORM DRUDE MODEL

We derive the dynamical fluctuation structure for a single parabolic conduction band with uniform electron density n and constant mobility $\mu_e = e\tau/m^*$, where τ is the spin-independent collision time and m^* the effective mass. The system is driven by a uniform field $\widetilde{\mathbf{E}} = \mathbf{E} = -E\hat{\mathbf{x}}$ acting in the negative (drain to source) direction. We take variations which are homogeneous over the sample region, so that the fluctuations of interest have no spatial dependence.

The Boltzmann equation in the model is

$$\left[\frac{\partial}{\partial t} + \frac{eE}{\hbar}\frac{\partial}{\partial k_x} + \frac{1}{\tau}\right] f_{\mathbf{k}}(t) = \frac{\langle f(t)\rangle}{\langle f^{\rm eq}\rangle}\frac{f_k^{\rm eq}}{\tau}. \tag{B1}$$

Since the Boltzmann operator is linear, the fluctuation structure is qualitatively similar to that for elastic scattering [differences arise from the inhomogeneous term in $f^{\rm eq}$, notably in the behaviours of $R(t)$ and $\Delta f(t)$]. We solve Eq. (B1) by Fourier transforms in reciprocal space, so that the transform $F_{\boldsymbol{\rho}} \equiv \Omega^{-1}\sum_{\mathbf{k}} f_{\mathbf{k}} \exp(i\mathbf{k}\cdot\boldsymbol{\rho})$ of the steady-state distribution takes the form

$$F_{\boldsymbol{\rho}} = \frac{F_{\mathbf{0}}}{F_0^{\rm eq}}\frac{F_\rho^{\rm eq}}{1 - ik_d\rho_x}, \tag{B2}$$

where $k_d = eE\tau/\hbar$ and $F_{\mathbf{0}} = \frac{1}{2}\langle f\rangle$ per spin state. Note that, while a formal distinction is made between $F_{\mathbf{0}}$ and $F_0^{\rm eq}$, the physical normalisation is always $F_{\mathbf{0}} = F_0^{\rm eq} = \frac{1}{2}n$.

The transform of the dynamic BGF, $\mathcal{R}_{\boldsymbol{\rho}\boldsymbol{\rho}'}(\omega) \equiv \Omega^{-2}\sum_{\mathbf{k},\mathbf{k}'} \mathrm{R}_{\mathbf{k}\mathbf{k}'}(\omega)\exp[i(\mathbf{k}\cdot\boldsymbol{\rho} - \mathbf{k}'\cdot\boldsymbol{\rho}')]$, has the equation

$$[-i\omega\tau - ik_d\rho_x + 1]\,\mathcal{R}_{\boldsymbol{\rho}\boldsymbol{\rho}'}(\omega) = \tau\delta(\boldsymbol{\rho} - \boldsymbol{\rho}') + \frac{\mathcal{R}_{\mathbf{0}\boldsymbol{\rho}'}(\omega)}{F_0^{\rm eq}}F_\rho^{\rm eq}. \tag{B3}$$

For $\boldsymbol{\rho} = \mathbf{0}$ this leads to

$$\mathcal{R}_{\mathbf{0}\boldsymbol{\rho}'}(\omega) = -\frac{\delta(\boldsymbol{\rho}')}{i(\omega + i\eta)}. \tag{B4}$$

On the other hand, the uncorrelated component of $\mathcal{R}_{\boldsymbol{\rho}\boldsymbol{\rho}'}$ scales with the steady-state solution $F_{\boldsymbol{\rho}}$ [in a collision-time model the asymptotic form $F_{\boldsymbol{\rho}}/\frac{1}{2}n$ replaces $\Delta F_{\boldsymbol{\rho}}/\frac{1}{2}\Delta n$]. Denoting the correlated part by $\mathcal{C}_{\boldsymbol{\rho}\boldsymbol{\rho}'}$ and recalling that the uncorrelated part exhausts the normalisation of $\mathcal{R}_{\mathbf{0}\boldsymbol{\rho}'}$, we obtain

$$\mathcal{R}_{\boldsymbol{\rho}\boldsymbol{\rho}'}(\omega) = \mathcal{C}_{\boldsymbol{\rho}\boldsymbol{\rho}'}(\omega) - \frac{\delta(\boldsymbol{\rho}')}{i(\omega + i\eta)}\frac{F_{\boldsymbol{\rho}}}{F_{\mathbf{0}}}. \tag{B5}$$

When the above is put together with Eqs. (B2)–(B4) we arrive, after some algebra, at the explicit formula for the correlated propagator:

$$\mathcal{C}_{\boldsymbol{\rho}\boldsymbol{\rho}'}(\omega) = \tau\frac{\delta(\boldsymbol{\rho} - \boldsymbol{\rho}') - \dfrac{F_{\boldsymbol{\rho}}}{F_{\mathbf{0}}}\delta(\boldsymbol{\rho}')}{1 - ik_d\rho_x - i\omega\tau}. \tag{B6}$$

We can use Eq. (B6) directly to evaluate both dissipative and non-dissipative contributions to the noise. Using the reciprocal-space representation $\mathbf{v} \leftrightarrow -i(\hbar/m^*)\partial/\partial\boldsymbol{\rho}$, the power density P of Eq. (60) is

$$P = 2\frac{e^2E^2}{k_BT}\left(-\frac{i\hbar}{m^*}\right)^2\left\{\frac{\partial}{\partial\rho_x}\int d^\nu\rho'\,\mathcal{C}_{\boldsymbol{\rho}\boldsymbol{\rho}'}(0)\frac{\partial}{\partial\rho'_x}\Delta F_{\rho'}^{\rm eq}\right\}_{\rho\to 0}$$

$$= 2\frac{e^2E^2\tau}{k_BT}\left(\frac{\hbar}{m^*}\right)^2\left\{-\frac{\partial^2}{\partial\rho_x^2}\Delta F_\rho^{\rm eq}\right\}_{\rho\to 0}$$

$$= \sigma E^2. \tag{B7}$$

The Drude conductivity $\sigma = ne\mu_e$ appears when we apply the relation $\{-\partial^2\Delta F_\rho^{\rm eq}/\partial\rho_x^2\}_{\rho\to 0} = \langle k_x^2\Delta f^{\rm eq}\rangle = m^*k_BTn/2\hbar^2$ to the middle line of the equation. A contribution containing $\langle v_x\Delta f^{\rm eq}\rangle = 0$ vanishes trivially.

The hot-electron spectral density S_g in the static limit [Eq. (64a)] is calculated similarly:

$$\begin{aligned}
S_g &= 2\frac{(e\mathbf{E}\cdot\hat{\mathbf{x}})^3}{\hbar}\left\{\int d^\nu\rho'\int d^\nu\rho'' v_x\mathcal{C}_{\boldsymbol{\rho\rho'}}(0)v'_x\mathcal{C}_{\boldsymbol{\rho'\rho''}}(0)(-i\rho''_x\Delta F_{\rho''}^{\rm eq})\right\}_{\rho\to 0} \\
&= 2\frac{e^3E^3\tau^2\hbar}{m^{*2}}\left\{\left[\frac{\partial}{\partial\rho_x}\frac{1}{1-ik_d\rho_x}\left(\frac{\partial}{\partial\rho_x}\frac{-i\rho_x\Delta F_\rho^{\rm eq}}{1-ik_d\rho_x}\right)\right]_{\rho\to 0}\right. \\
&\quad\left.-\left[\frac{\partial}{\partial\rho_x}\frac{F_{\boldsymbol\rho}/F_{\mathbf{0}}}{1-ik_d\rho_x}\right]_{\rho\to 0}\left[\frac{\partial}{\partial\rho'_x}\frac{-i\rho'_x\Delta F_{\rho'}^{\rm eq}}{1-ik_d\rho'_x}\right]_{\rho'\to 0}\right\}.
\end{aligned} \tag{B8}$$

We evaluate this with the help of the relations $\Delta F_0^{\rm eq} = \frac{1}{2}\Delta n$ and $\{\partial F_{\boldsymbol\rho}/\partial\rho_x\}_{\rho\to 0} = ik_dF_{\mathbf{0}}$, the latter following from Eq. (B2). The result is

$$S_g = \sigma m^*\mu_e^2E^4\frac{\Delta n}{n}. \tag{B9}$$

APPENDIX C: WEAKLY NONUNIFORM DRUDE MODEL

We derive the spatio-temporal correlations within the Drude model of the preceding Appendix. The problem is to calculate the propagation of a single electron added to N uniformly distributed electrons at a *specific* point in the sample at $t = 0$. This constitutes a weak inhomogeneity.

Since the scattering is spin-independent, we consider a zero-spin model with effective density $\frac{1}{2}n$. The equation of motion for the dynamical propagator in the frequency domain is

$$\left[\tau\mathbf{v_k}\cdot\frac{\partial}{\partial\mathbf{r}} + k_d\frac{\partial}{\partial k_x} + 1 - i\omega\tau\right]\mathrm{R}_{\alpha\alpha'}(\omega) = \tau\delta_{\mathbf{rr'}}\delta_{\mathbf{kk'}} + \frac{\sum_{\alpha''}\mathrm{R}_{\alpha''\alpha'}(\omega)}{\frac{1}{2}N}f_k^{\rm eq}; \tag{C1}$$

Now define the Fourier transform of the propagator,

$$\mathrm{R}_{\mathbf{kk'}}(\mathbf{q},\mathbf{q}',\omega) = \int_\Omega d^\nu r\int_\Omega d^\nu r'\ \mathrm{R}_{\alpha\alpha'}(\omega)\exp[-i(\mathbf{q}\cdot\mathbf{r} - \mathbf{q}'\cdot\mathbf{r}')]. \tag{C2}$$

Eq. (C1) becomes

$$\begin{aligned}
\left[k_d\frac{\partial}{\partial k_x} + \frac{i\hbar\tau}{m^*}\mathbf{q}\cdot\mathbf{k} + 1 - i\omega\tau\right]\mathrm{R}_{\mathbf{kk'}}(\mathbf{q},\mathbf{q}',\omega) &= \tau\Omega\delta_{\mathbf{kk'}}\Omega\varphi_\nu(\mathbf{q}-\mathbf{q}') \\
&\quad + \langle\mathrm{R}(\mathbf{0},\mathbf{q}',\omega;\mathbf{k}')\rangle\Omega\varphi_\nu(\mathbf{q})\frac{f_k^{\rm eq}}{\frac{1}{2}N},
\end{aligned} \tag{C3}$$

where

$$\varphi_\nu(\mathbf{q}) = \frac{1}{\Omega}\int_\Omega d^\nu r e^{-i\mathbf{q}\cdot\mathbf{r}}. \tag{C4}$$

For a three-dimensional system with cylindrical symmetry about the x-axis we write $\mathbf{q} = (q_x, \mathbf{q}_\perp)$ and $\Omega = \pi R^2L_x$ where L_x is the sample length and R its radius. The function φ_3 can be written as

$$\varphi_3(\mathbf{q}) = \varphi_1(q_x;\tfrac{1}{2}L_x)\varphi_2(\mathbf{q}_\perp;R) \equiv \frac{\sin(\frac{1}{2}L_xq_x)}{\frac{1}{2}L_xq_x}\frac{J_1(Rq_\perp)}{\frac{1}{2}Rq_\perp} \tag{C5}$$

with $J_1(u)$ the first-order Bessel function. For a two-dimensional strip of width $2R$, the role of φ_3 is assumed by the product $\varphi_1(q_x; \frac{1}{2}L_x)\varphi_1(q_\perp; R)$.

The correlated dynamical propagator C associated with R has the form

$$\begin{aligned}
\mathrm{C}_{\mathbf{kk}'}(\mathbf{q},\mathbf{q}',\omega) &= \mathrm{R}_{\mathbf{kk}'}(\mathbf{q},\mathbf{q}',\omega) - \langle \mathrm{R}(\mathbf{0},\mathbf{q}',\omega;\mathbf{k}')\rangle \varphi_\nu(\mathbf{q})\frac{f_{\mathbf{k}}}{\frac{1}{2}n} \\
&= \mathrm{R}_{\mathbf{kk}'}(\mathbf{q},\mathbf{q}',\omega) + \frac{\Omega\varphi_\nu(\mathbf{q})\varphi_\nu(\mathbf{q}')}{i(\omega+i\eta)}\frac{f_{\mathbf{k}}}{\frac{1}{2}n},
\end{aligned} \tag{C6}$$

wherein $\langle \mathrm{R} \rangle$ is evaluated by summing over $\mathbf{k}$ on both sides of Eq. (C3) in the limit $q \to 0$. The equation of motion for C is[19]

$$\begin{aligned}
&\left[k_d \frac{\partial}{\partial k_x} + \frac{i\hbar\tau}{m^*}\mathbf{q}\cdot\mathbf{k} + 1 - i\omega\tau \right] \mathrm{C}_{\mathbf{kk}'}(\mathbf{q},\mathbf{q}',\omega) \\
&= \tau\Omega^2 \delta_{\mathbf{kk}'}\varphi_\nu(\mathbf{q}-\mathbf{q}') + \langle \mathrm{R}(\mathbf{0},\mathbf{q}',\omega;\mathbf{k}')\rangle\varphi_\nu(\mathbf{q})\frac{f_k^{\mathrm{eq}}}{\frac{1}{2}n} \\
&\quad - \langle \mathrm{R}(\mathbf{0},\mathbf{q}',\omega;\mathbf{k}')\rangle\varphi_\nu(\mathbf{q})\left[k_d\frac{\partial}{\partial k_x} + 1 - i\omega\tau \right]\frac{f_{\mathbf{k}}}{\frac{1}{2}n} \\
&= \tau\Omega\left(\Omega\delta_{\mathbf{kk}'}\varphi_\nu(\mathbf{q}-\mathbf{q}') - \varphi_\nu(\mathbf{q})\varphi_\nu(\mathbf{q}')\frac{f_{\mathbf{k}}}{\frac{1}{2}n} \right).
\end{aligned} \tag{C7}$$

Terms $\sim 1/\omega$, singular in the static limit, cancel identically by the fact that f is the Boltzmann solution for the uniform steady state. Equation (C7) is solved at arbitrary fields with the integrating factor[30]

$$X_{\mathbf{k}}(\mathbf{q},\omega) = \exp\left\{ \frac{k_x}{k_d}\left[\frac{i\hbar\tau}{m^*}\left(\frac{q_x k_x}{2} + \mathbf{q}_\perp\cdot\mathbf{k}_\perp \right) + 1 - i\omega\tau \right] \right\}; \tag{C8}$$

in the low-field limit, the non-analytic character of X is clear from the occurrence of $1/k_d \propto 1/E$ in its exponent. Using X we first generate the auxiliary propagator $\mathrm{C}^{(0)}$ satisfying

$$\left[k_d \frac{\partial}{\partial k_x} + \frac{i\hbar\tau}{m^*}\mathbf{q}\cdot\mathbf{k} + 1 - i\omega\tau \right] \mathrm{C}^{(0)}_{\mathbf{kk}'}(\mathbf{q},\omega) = \tau\Omega\delta_{\mathbf{kk}'}. \tag{C9a}$$

The expression for $\mathrm{C}^{(0)}$ is

$$\mathrm{C}^{(0)}_{\mathbf{kk}'}(\mathbf{q},\omega) = \frac{\tau\Omega}{k_d L_x}\delta_{\mathbf{k}_\perp\mathbf{k}'_\perp}\theta(k_x - k'_x)X^{-1}_{\mathbf{k}}(\mathbf{q},\omega)X_{\mathbf{k}'}(\mathbf{q},\omega), \tag{C9b}$$

which furnishes the complete solution to Eq. (C7) as

$$\mathrm{C}_{\mathbf{kk}'}(\mathbf{q},\mathbf{q}',\omega) = \Omega\varphi_\nu(\mathbf{q}-\mathbf{q}')\mathrm{C}^{(0)}_{\mathbf{kk}'}(\mathbf{q},\omega) - \frac{\varphi_\nu(\mathbf{q})\varphi_\nu(\mathbf{q}')}{\frac{1}{2}n}\sum_{\mathbf{k}''}\mathrm{C}^{(0)}_{\mathbf{kk}''}(\mathbf{q},\omega)f_{\mathbf{k}''}. \tag{C10}$$

For our study of shot noise it is convenient to consider the propagator integrated over $\mathbf{q}'$, namely

$$\mathrm{C}^{(b)}_{\mathbf{kk}'}(\mathbf{q},\omega) = \int \frac{d^\nu q'}{(2\pi)^\nu}\mathrm{C}_{\mathbf{kk}'}(\mathbf{q},\mathbf{q}',\omega), \tag{C11}$$

equivalent to the bulk solution in an infinitely wide conductor, $R \to \infty$. The form of $\mathrm{C}^{(b)}$ follows from Eq. (C10). It is

$$\mathrm{C}^{(b)}_{\mathbf{kk}'}(\mathbf{q},\omega) = \mathrm{C}^{(0)}_{\mathbf{kk}'}(\mathbf{q},\omega) - \frac{\varphi_\nu(\mathbf{q})}{\frac{1}{2}n\Omega}\sum_{\mathbf{k}''}\mathrm{C}^{(0)}_{\mathbf{kk}''}(\mathbf{q},\omega)f_{\mathbf{k}''}. \tag{C12}$$

In the zero-field limit this goes to

$$\mathrm{C}^{(b)}_{\mathbf{k}\mathbf{k}'}(\mathbf{q},\omega)\Big|_{E\to 0} = \frac{\Omega\delta_{\mathbf{k}\mathbf{k}'} - \varphi_\nu(\mathbf{q})\dfrac{f^{\rm eq}_k}{\frac{1}{2}n}}{\dfrac{i\hbar}{m^*}\mathbf{q}\cdot\mathbf{k} + \tau^{-1} - i\omega}. \qquad \text{(C13)}$$

Finally, for reference, we also record the expression for the uniform distribution $\Delta f_{\mathbf{k}}$, needed in the shot-noise application:

$$\Delta f_{\mathbf{k}} = \int_{-\infty}^{k_x} \frac{dk'_x}{k_d} e^{-(k_x - k'_x)/k_d} \Delta f^{\rm eq}_{k'}. \qquad \text{(C14a)}$$

In the degenerate limit this becomes

$$\Delta f_{\mathbf{k}} = \frac{m^* k_B T}{\hbar^2 p_\perp k_d}\theta(k_F - k_\perp)\Big[\theta(k_x - p_\perp)e^{(p_\perp - k_x)/k_d} + \theta(k_x + p_\perp)e^{-(p_\perp + k_x)/k_d}\Big], \qquad \text{(C14b)}$$

where $p_\perp = (k_F^2 - k_\perp^2)^{\frac{1}{2}}$. In one dimension, set $k_\perp = 0$.

---

† Electronic address: Fred.Green@tip.csiro.au

‡ Electronic address: mukunda.das@anu.edu.au

[1] We refer to calculations based on the microscopics of degenerate systems rather than predominantly numerical methods such as Monte Carlo; see for example P. Tadyszak, F. Danneville, A. Cappy, L. Reggiani, L. Varani, and L. Rota, Appl. Phys. Lett. **69**, 1450 (1996).

[2] D. Pines and P. Nozières, *The Theory of Quantum Liquids* (Benjamin, New York, 1966).

[3] N. W. Ashcroft and N. D. Mermin, *Solid State Physics* (Holt, Rinehart, and Winston, New York, 1976).

[4] M. Reznikov, M. Heiblum, H. Shtrikman, and D. Mahalu, Phys. Rev. Lett. **75**, 3340 (1995).

[5] A. Kumar, L. Saminadayar, D. C. Glattli, Y. Jin, and B. Etienne, Phys. Rev. Lett. **76**, 2778 (1996).

[6] A. H. Steinbach, J. M. Martinis, and M. H. Devoret, Phys. Rev. Lett. **76**, 3806 (1996).

[7] R. J. Schoelkopf, P. J. Burke, A. A. Kozhevnikov, D. E. Prober, and M. J. Rooks, Phys. Rev. Lett. **78**, 3370 (1997).

[8] H. Pothier, S. Guéron, N. O. Birge, D. Esteve, and M. H. Devoret, Phys. Rev. Lett. **79**, 3490 (1997).

[9] R. de Picciotto, M. Reznikov, M. Heiblum, V. Umansky, G. Bunin, and D. Mahalu, Nature **389**, 162 (1997).

[10] L. Saminadayar, D. C. Glattli, Y. Jin, and B. Etienne, Phys. Rev. Lett. **79**, 2526 (1997).

[11] For a comprehensive overview, see M. J. M. de Jong and C. W. J. Beenakker in *Mesoscopic Electron Transport*, edited by L. P. Kouwenhoven, G. Schön, and L. L. Sohn, NATO ASI Series E (Kluwer Academic, Dordrecht, 1997).

[12] G. B. Lesovik, JETP Lett. **66**, 1243 (1989).

[13] M. Büttiker, Phys. Rev. Lett. **65**, 2901 (1990); Phys. Rev. B **46**, 12485 (1992).

[14] C. W. J. Beenakker and M. Büttiker, Phys. Rev. B **46**, 189 (1992).

[15] Th. Martin and R. Landauer, Phys. Rev. B **45**, 1742 (1992).

[16] K. E. Nagaev, Phys. Lett. A **169**, 103 (1992); Phys. Rev. B **52**, 4740 (1994).

[17] M. J. M. de Jong and C. W. J. Beenakker, Phys. Rev. B **51** 16867 (1995).

[18] R. Landauer, IBM J. Res. Dev. **1**, 233 (1957).

[19] Sh. M. Kogan and A. Ya. Shul'man, Zh. Eksp. Teor. Fiz. **56**, 862 (1969) [Sov. Phys. JETP **29**, 467 (1969)].

[20] The transition from coherent to incoherent prescriptions for noise has been studied by A. Shimizu and M. Ueda, Phys. Rev. Lett. **69**, 1403 (1992).

[21] R. Landauer, Phys. Rev. B **47**, 16427 (1993).

[22] R. C. Liu, P. Eastman, and Y. Yamamoto, Solid State Commun. **102**, 785 (1997).

[23] R. Landauer, Physica (Amsterdam) **227B**, 156 (1996).

[24] D. K. Ferry and S. M. Goodnick, *Transport in Nanostructures* (Cambridge University Press, Cambridge, U. K., 1997).

[25] C. J. Stanton and J. W. Wilkins, Phys. Rev. B **35**, 9722 (1987); Phys. Rev. B **36**, 1686 (1987).

[26] B. L. Altshuler, L. S. Levitov, and A. Yu. Yakovets, Pis'ma Zh. Eksp. Teor. Fiz. **59**, 821 (1994) [JETP Lett. **59**, 857 (1994)].

[27] T. González, C. González, J. Mateos, D. Pardo, L. Reggiani, O. M. Bulashenko, and J. M. Rubí, Phys. Rev. Lett. **80**, 2901 (1998).

[28] G. Rickayzen, *Green's Functions and Condensed Matter* (Academic Press, London, 1980).

[29] S. V. Gantsevich, V. L. Gurevich, and R. Katilius, Riv. Nuovo Cimento **2**, 1 (1979).

[30] C. J. Stanton, Ph.D. thesis, 1986 (Cornell University, unpublished).

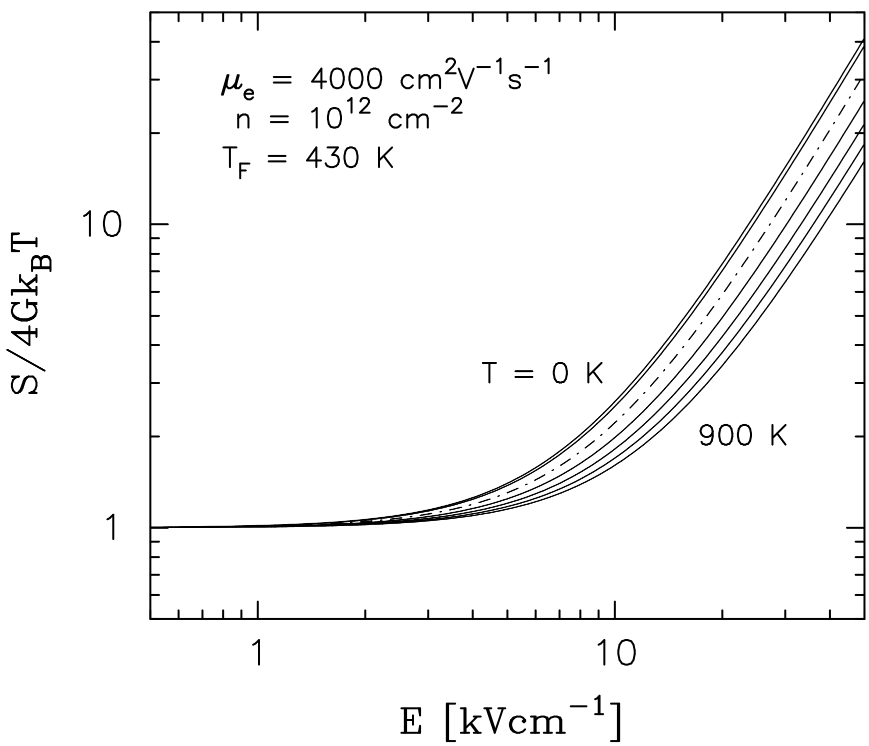
[31] F. Green and M. J. Chivers, Phys. Rev. B **54**, 5791 (1996).
[32] Y. Naveh, D. V. Averin, and K. K. Likharev, Phys. Rev. Lett. **79**, 3482 (1997).
[33] T. González, O. M. Bulashenko, J. Mateos, D. Pardo, and L. Reggiani, Phys. Rev. B **56**, 6424 (1997); O. M. Bulashenko, J. Mateos, D. Pardo, T. González, L. Reggiani, and J. M. Rubí, Phys. Rev. B **57**, 1366 (1998).
[34] K. E. Nagaev, Phys. Rev. B **57**, 4628 (1998).
[35] C. M. Van Vliet, IEEE Trans. Electron Devices **41**, 1902 (1994).
[36] J. P. Nougier, in *Physics of Nonlinear Transport in Semiconductors*, edited by D. K. Ferry, J. R. Barker, and C. Jacoboni (Plenum, New York, 1980), p. 415 ff.; see also J. P. Nougier, IEEE Trans. Electron Devices **41**, 2034 (1994).
[37] L. Reggiani, P. Lugli, and V. Mitin, Phys. Rev. Lett. **60**, 736 (1988).
[38] The Poisson equation is always defined in three dimensions. To interpret Eq. (3) appropriately when $\nu < 3$, the electron density $\langle f \rangle$ must be understood to carry a (separable) factor in the $3-\nu$ transverse space co-ordinates. Thus, for a transport problem confined strictly to two dimensions, the Poisson source term contains $\langle f(\mathbf{r}) \rangle \equiv \delta(z) \langle f(\mathbf{r}_\perp) \rangle$, where z is orthogonal to the plane $(\mathbf{r}_\perp; z = 0)$. On the other hand, the stabilising background distribution $n^+(\mathbf{r})$ can be fully three-dimensional, as in a modulation-doped heterostructure.[24]
[39] F. Green, Phys. Rev. B **54**, 4394 (1996).
[40] D. K. C. MacDonald, *Noise and Fluctuations: an Introduction* (Wiley, New York, 1962).
[41] F. Green and M. P. Das, cond-mat/9709142.
[42] P. M. Bakshi and E. P. Gross, Ann. Phys. **24**, 419 (1968).
[43] R. Kubo, M. Toda, and N. Hashitsume, *Statistical Physics II: Nonequilibrium Statistical Mechanics* (2nd ed., Springer, Berlin, 1991), pp 199 and 200.
[44] The BGF description does produce a smooth crossover from thermal to shot-noise-*like* behaviour in some regimes, but always scaled by the degeneracy factor $\Delta n/n$. See our Ref. 41; for the classical limit see Stanton, Ref. 30.
[45] L. P. Kadanoff and G. Baym, *Quantum Statistical Mechanics* (Benjamin, Reading, MA, 1962).
[46] Sh. M. Kogan, Phys. Rev. A **44**, 8072 (1991).

FIG. 1. Zero-frequency spectral density of nonequilibrium thermal noise in a uniform, two-dimensional electron gas in GaAs, plotted as a function of the external field from $T = 0$ to 900 K in 150 K steps. Normalisation is to the equilibrium Johnson-Nyquist value. At low temperature, degeneracy sets the scale of the contribution from nonequilibrium electron heating. At high temperature, the hot-electron component shifts up towards higher fields as the equilibrium component gains dominance. Dot-dashed line: thermal noise at 300 K.

FIG. 2. (a) Shot noise in the Drude model of a degenerate one-dimensional wire, as a function of current in units of the Fermi current and normalised to full shot noise. Each curve is for a fixed ratio λ/l of mean free path to length of wire; the same set of ratios is used for all subsequent figures. At high currents and in the ballistic limit (upper curves), the shot noise tends to its full value. At low currents and away from the ballistic limit, degeneracy inhibits the natural tendency of the shot noise to exponential suppression with increased sample length. (b) Full line: shot noise of classical carriers, as a function of current in units of the thermal current. The attenuation at low currents is much more pronounced than in (a). Dots: high-field asymptote defined by Eq. (75). Note how both classical and fermionic results rapidly assume this form at higher currents.

FIG. 3. Shot noise in the degenerate Drude model for (a) very wide two-dimensional strips, and (b) three-dimensional cylinders of very large radius. The behaviour at low currents differs significantly from Fig. 2(a), with progressively greater suppression at longer wire lengths and higher dimensionality. At high currents the asymptotic behaviour is identical with that in one dimension.

FIG. 4. Shot noise in the degenerate Drude model for (a) narrow strips, and (b) thin cylinders. Away from the ballistic limit (topmost curves) there is remarkable shot-noise suppression over the entire range of the current. This effect is inherent in the kinematic term of the higher-dimensional Boltzmann equation; it cannot be simulated by a one-dimensional approximation.



μ_e = 4000 cm^{2}V^{-1}s^{-1}
n = 10^{12} cm^{-2}
T_F = 430 K
T = 0 K
900 K
10
1
1
10
S/4Gk$_B$T
E [kVcm^{-1}]

FIG. 1

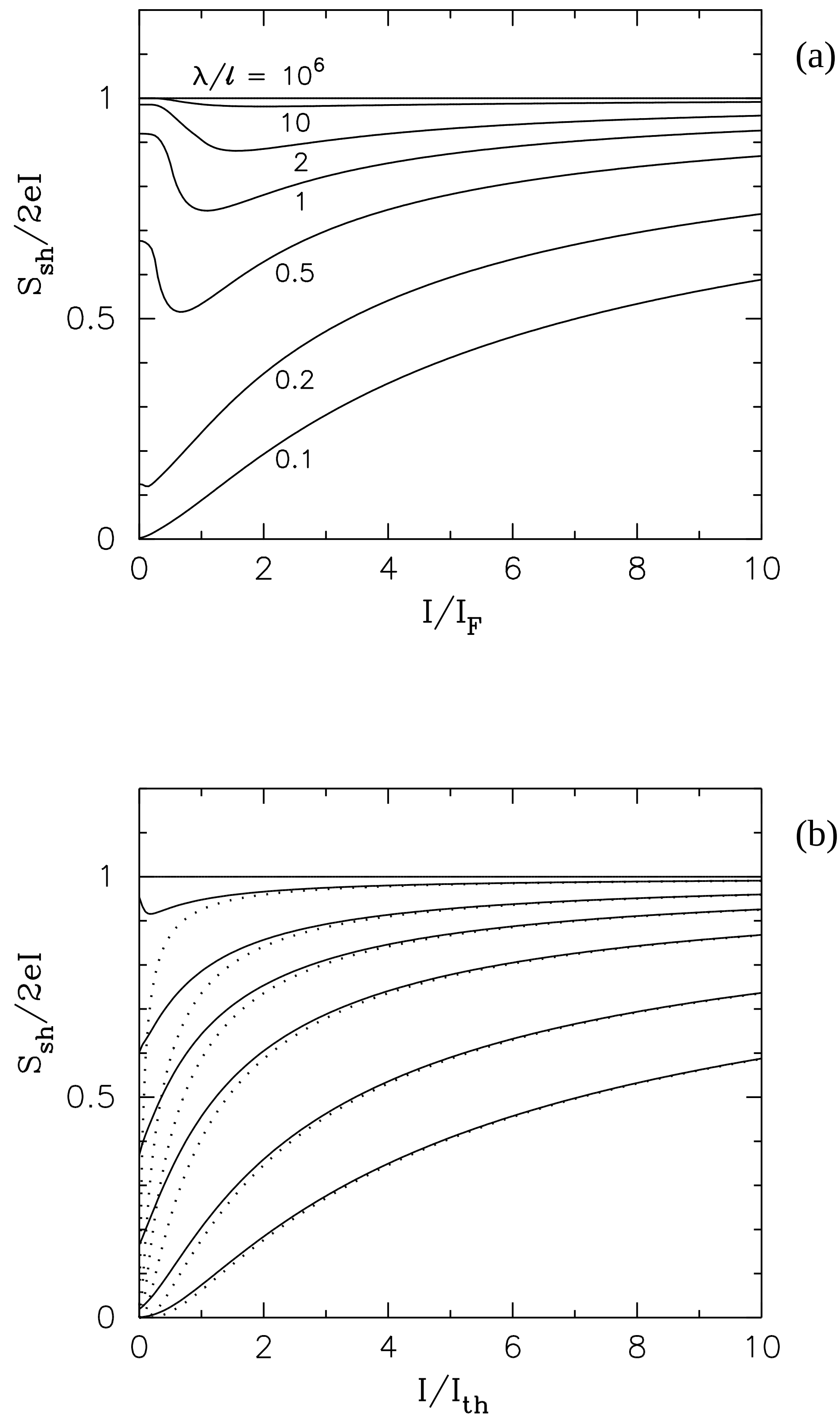
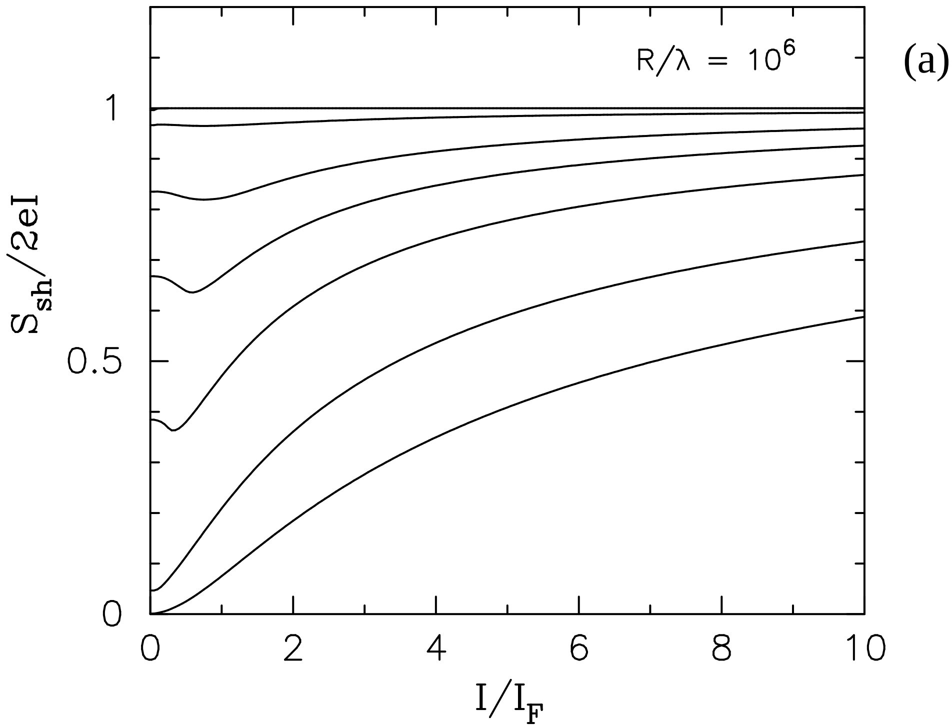
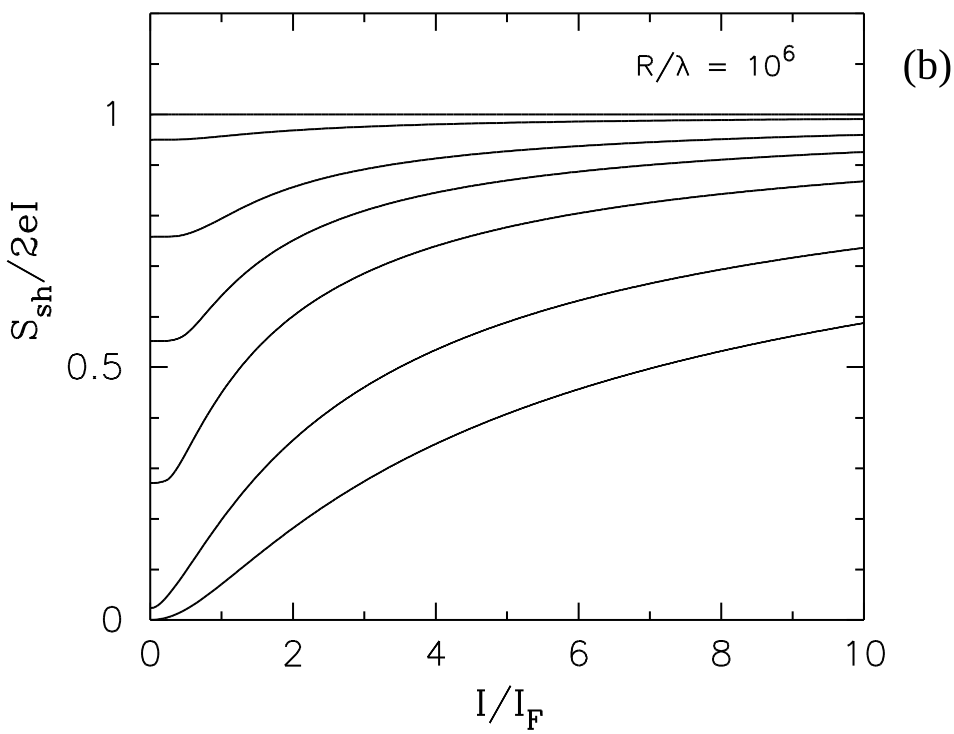

(a)
λ/l = 10^6
10
2
1
0.5
0.2
0.1
$S_{sh}/2eI$
I/I_F
0
0.5
1
0
2
4
6
8
10
(b)
$S_{sh}/2eI$
I/I_{th}
0
0.5
1
0
2
4
6
8
10

FIG. 2

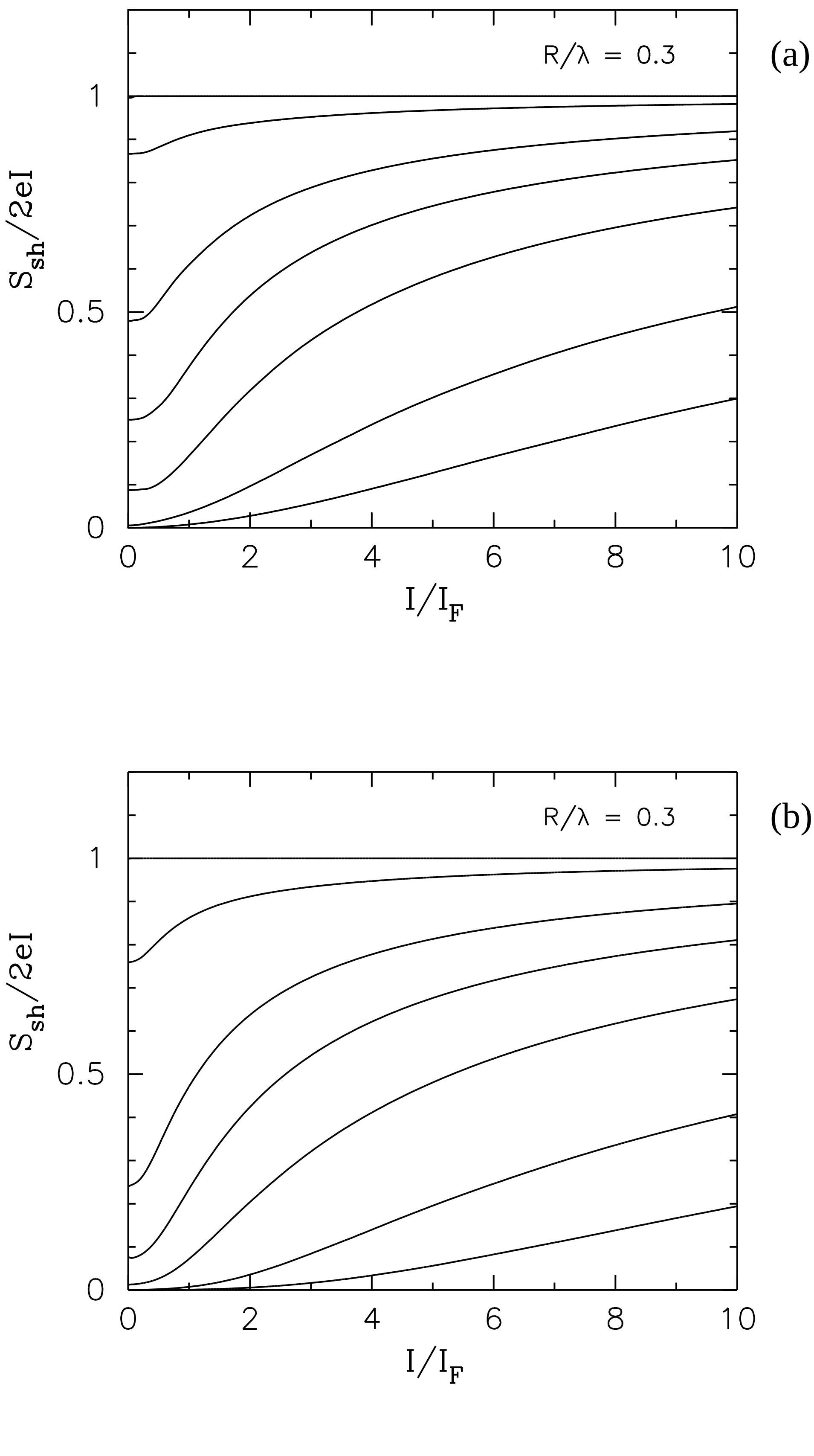


(a)
$R/\lambda = 10^6$
$S_{sh}/2eI$
I/I_F
0
0.5
1
0
2
4
6
8
10



(b)
$R/\lambda = 10^6$
$S_{sh}/2eI$
I/I_F
0
0.5
1
0
2
4
6
8
10

FIG. 3



(a)
R/λ = 0.3
$S_{sh}/2eI$
0
0.5
1
0
2
4
6
8
10
I/I_F
(b)
R/λ = 0.3
$S_{sh}/2eI$
0
0.5
1
0
2
4
6
8
10
I/I_F

FIG. 4